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IS CONSUMPTION INSUFFICIENTLY SENSITIVE
TO INNOVATIONS IN INCOME?

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ABSTRACT

Deaton (1986) has noted that if income is a first-order autoregressive process in first differences, then a simple version of Friedman's permanent income hypothesis (SPIH) implies that measured U.S. consumption is insufficiently sensitive to innovations in income. This paper argues that this implication of the SPIH is a consequence of the fact that it ignores the role of the substitution effect in the consumption decision. Using a parametric version of the standard model of economic growth, the paper shows that very small movements in interest rates are sufficient to induce an empirically plausible amount of consumption smoothing. Since an overall evaluation of the model's explanation for the observed smoothness of consumption requires examining its implications for other aspects of the data, the paper also explores some of these.

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A basic fact about U.S. macroeconomic data is that consumption is a much smoother time series than income. A classic explanation for this is a simple version of Friedman's permanent income hypothesis (SPIH). According to the SPIH, an innovation to current income causes households to revise their consumption plan by the annuity value of that innovation.^{1/} The annuity value is computed under the assumption of a constant interest rate.^{2/} When U.S. income data are modeled as the sum of a linear trend and a covariance stationary process, then innovations to income do not affect the long-run income outlook and their annuity value is smaller than the innovation itself [Deaton, 1986, eqns. (7)-(8)]. Thus, with this model of income the SPIH predicts, correctly, that consumption is smoother than income in the sense that consumption's innovations are a fraction of income's.

However, a growing number of researchers are attracted to the view that U.S. income data can be represented as a positively autocorrelated process in first differences. In this view, an innovation to income produces a change in the long-run outlook for income and has an annuity value greater than the innovation itself. Then, the SPIH has the strongly counterfactual implication that consumption is less smooth than income. Put differently, measured U.S. consumption is insufficiently sensitive to innovations in income, relative to the SPIH and a first-difference specification for income. This is an implication emphasized by Deaton (1986).

The analysis below suggests that the reason for the SPIH's counterfactual implication is its fixed interest rate assumption. Using a parametric version of the standard model of economic growth, in which the log of income has a unit autoregressive root, I show that very small movements in interest rates are enough to induce an empirically plausible amount of consumption smoothing.

The particular model I study is G. Hansen's (1985), modified so that it displays growth on average and variables like the log of consumption and of income are difference stationary processes. (Another model that formalizes the argument in this paper is Christiano, Eichenbaum, and Marshall's, 1987.) In this model, disturbances to output are generated by marginal productivity shocks, so that a jump in output produces not only an income effect--as the SPIH emphasizes--but also a small substitution effect. I find that the smoothness of consumption relative to income implied by this model is about what is observed. This is noteworthy because the model's parameter values are chosen to match averages of U.S. time series data, with second-moment properties playing a minimal role. A serious evaluation of the model's explanation for consumption smoothing cannot ignore its implications for other aspects of the data. I therefore also explore some of these.

I. The Model

At date $t = 0$, a representative agent chooses decision rules for c_t , h_t , and dk_t to maximize^{3/}

$$(1) \quad E_0 \sum_{t=0}^{\infty} \beta^t \{ \ln c_t - \gamma h_t \}, \quad \text{for } \gamma > 0$$

subject to the technology

$$(2) \quad c_t + k_t - [(1-\delta)/n]k_{t-1} = n^{-\theta} (z_t h_t)^{(1-\theta)} k_{t-1}^{\theta}.$$

Here c_t denotes consumption, h_t hours worked, k_t end-of-quarter stock of capital, and dk_t capital investment in quarter t . The expression on the right side of (2) is output y_t . This is assumed to be related to k_{t-1} , h_t , and a technology shock z_t by a Cobb-Douglas production function. The variables k_t and dk_t are related by $k_t - [(1-\delta)/n]k_{t-1} \equiv dk_t$. All variables are measured per capita. Assumed constant are the parameters n , the gross growth rate of the population, and δ , the rate of depreciation of a unit of capital.

The growth rate x_t of the technology shock is assumed to be covariance stationary with a first-order autoregressive structure. In particular,

$$(3) \quad z_t = z_{t-1} \exp(x_t), \quad x_t = \mu + \rho x_{t-1} + \varepsilon_t, \quad \text{for } |\rho| < 1$$

where ε_t is white noise. According to (3), the average growth rate of the technology shock is $\mu/(1-\rho)$, with first-order autocorrelation ρ . In the model, c_t , y_t , k_t , and dk_t grow at the same rate as z_t . On average, per capita hours h_t do not grow, which is roughly in accord with postwar U.S. experience.

To derive the model's implications for the stochastic properties of its endogenous variables, the decision rules are needed. Because obtaining these exactly is complicated, instead I obtain approximations. (For details, see Christiano, 1986b.)

The analysis also requires the equilibrium rate r_t at which a unit of consumption can be transformed risklessly from t to $t + 1$. This is defined as

$$(4) \quad 1 + r_t = \frac{\partial u(c_t, h_t) / \partial C_t}{\beta E_t \partial u(c_{t+1}, h_{t+1}) / \partial C_{t+1}}$$

Here $u(c_t, h_t) = \log(c_t) - \gamma h_t$ and $C_t \equiv N_t c_t$, where N_t is the population in quarter t . When σ_ε is small, the average value of r_t is $[\mu/(1-\rho)](n/\beta) - 1$. This is roughly the sum of the economy-wide rate of consumption growth $[\mu/(1-\rho) - 1]$ and the subjective rate of time discount $(\beta^{-1} - 1)$.

II. Parameter Values

To deduce the model's quantitative implications, values must be assigned to its parameters. I choose these:^{4/} $\rho = -.077$, $\mu = .0035$, $\gamma = .0026$, $n = 1.00324$, $\beta = .99$, $\delta = .018$, $\theta = .39$, and $\sigma_\varepsilon = .019$. The value of n is the average quarterly growth in the quality-adjusted, working-age population in 1952-84. With this value, $\delta = .018$ is required if the gross investment series implied by dk_t is to resemble the gross investment series published by the U.S. Department of Commerce. The value of β is from Kydland and Prescott (1982). Values for θ , γ , and $\mu/(1-\rho)$ are chosen to roughly match the model's implications for the average values of h_t , c_t/y_t , and k_t/y_t with their empirical counterparts in U.S. data for 1956.2-1984.1. The implied averages (and empirical values) for these variables are 323.9 (320.4), .72 (.72), and 11.32 (10.58), respectively. The values of ρ , μ , and σ_ε are based on an analysis of the time series properties of z_t , which can be

measured using data on y_t , k_t , and h_t given the values assigned to θ and n .

Consumption is defined as public and private consumption of goods, services, and the services of the stock of durables. The stock of capital is defined as the stock of public and private equipment and structures plus the stock of consumer durables plus public and private residential capital. Capital investment is defined to conform to the definition of the capital stock. I use G. Hansen's (1984) time series on hours worked measured in efficiency units. Variables are converted to per capita terms by the working-age population, measured in efficiency units. The risk-free rate is proxied by the ex post real return on three-month U.S. Treasury bills. (For further details on the data and this methodology for choosing parameter values, see Christiano, 1986a.)

III. Relative Smoothness of Consumption

Here I describe the dynamic properties of the model from two perspectives. First, the model's shock response function is used to deduce the model's implication for the relative smoothness of consumption and income. Then several of the model's unconditional second-moment properties are examined. These provide an alternative, complementary, measure of smoothness.

Figure 1 shows the first 30 quarters' responses of c_t , h_t , dk_t , and y_t to a one standard deviation innovation in the growth rate of the technology shock z_t in period 2 given that the system is on a steady-state growth path in $t = 0, 1$ (that is, $\varepsilon_2 = .019$, $\varepsilon_t = 0$ for $t = 0, 1, 3, 4, \dots$). The curves are the quar-

terly percentage deviations in these variables from a baseline scenario in which $\varepsilon_t = 0$ for $t = 0, 1, 2, 3, \dots$

With the assumed stochastic structure of z_t , an innovation to z_t is 92.85 percent $[100/(1-\rho)]$ permanent. Thus, in period 2, z_t jumps 1.9 percent above its baseline growth path, then declines to a path 1.76 percent above the baseline. After the shock, all the model's variables except r_t and h_t end up 1.76 percent above the baseline. As Figure 1 shows, consumption rises only gradually to this higher growth path. In particular, households choose not to adjust consumption immediately, as the SPIH-- which only recognizes an income effect--implies. This reflects households' desire to delay consumption when the return to investment is high (the substitution effect). Thus, capital investment responds strongly.

On Figure 1, note the early spikes in the responses of dk_t , h_t , and y_t . This reflects the fact that 7.15 percent of the initial 1.9 percent jump in z_t is only temporary. The lack of a spike in c_t reflects the small response of consumption to a temporary disturbance, which explains the pronounced spike in capital investment. Hours also respond fairly strongly to the temporary component in the productivity shock (as in G. Hansen, 1985).

The ratio of the jumps in consumption and income in period 2 is .32; that is, consumption's innovation is about 32 percent of income's. The empirically measured value of this ratio is 33 percent.^{5/}

The shock response of r_t is not on Figure 1 because it is so small. In the steady state, $r_t = 1.01667$. After the shock, r_t rises to 1.01728, then declines monotonically back to 1.01667. Thus, the effect on the interest rate is a negligible six one-hundredths of a basis point. Evidently, this model generates an empirically plausible degree of smoothness in consumption with only very little variation in the interest rate.

Next, I report smoothness properties of the model based on unconditional second moments. I refer to these measures of smoothness as volatility. Table 1 reports measures of the volatility of c_t , dk_t , h_t , and r_t relative to that of y_t as well as the volatility of y_t itself. The volatility of c_t , dk_t , and y_t is the standard deviation of the log of the first difference of these variables. The volatility of h_t and r_t is the standard deviation of their levels. The relative volatility measures are the ratios of these to the volatility of y_t . All standard deviations are computed for variables predicted by the model to be covariance stationary. Means and standard deviations for the volatility measures are computed by simulating 1,000 sets of 112 observations from the model. In each simulation, the decision rules of the model are solved with initial conditions on a steady-state growth path and ε_t 's drawn independently from a normal random number generator with mean zero and standard error .019.

Table 1 also shows empirical estimates of the volatility measures for the U.S. economy. Note that consumption's relative volatility is .49. This is quite close to the model's prediction,

which is only about 1.6 standard deviations lower. Thus--relative to this model, but in striking contrast to the SPIH--if there is a puzzle it is that the empirical relative variability of consumption is too high, not too low.

IV. Other Implications of the Model

The model does less well on other dimensions. Table 1 shows that the empirical measures for both investment and hours, for example, are more than three standard deviations from the model's predictions.

The most substantial evidence in Table 1 of a mismatch between the model's implications and the data is that for the risk-free return. The empirical measure of its relative variability is 110.32 standard deviations higher than the model's prediction. Also, the empirical correlation between the risk-free rate and consumption growth is 14.45 standard deviations below that correlation in the model, and the empirical average of the risk-free rate is 16.07 standard deviations below the model's average. It is not clear whether these discrepancies reflect shortcomings of the model or of my empirical measure of the risk-free rate.^{6/}

Another thing the model explains less well is the serial correlation in Δc_t and Δy_t . [$\Delta u_t \equiv \log(u_t) - \log(u_{t-1})$.] For example, as expected given the small movements in r_t , Δc_t is virtually uncorrelated with lagged Δc_t , Δy_t , Δdk_t , r_t , and h_t . The corresponding empirical quantities are all larger. (Only the correlation with lagged Δc_t is in Table 1.) What is perhaps more

surprising is that the serial correlation properties of y_t closely match those of z_t with capital accumulation seemingly playing a small role. Thus, the model's first-order autocorrelation of Δy_t is $-.119$, or 5.34 standard deviations below the empirical value. Not surprisingly, part of the reason this model can match the observed relative smoothness of consumption is this negative serial correlation. For example, with $\rho = .2$ but all other parameters, including $\mu/(1-\rho)$, unchanged, the first-order serial correlation of Δy_t averages the empirically plausible $.349$, with standard deviation $.084$. Here, however, a consumption innovation is 51 percent of an income innovation and the volatility of consumption is 66 percent that of income, with standard deviation $.038$. Although these numbers are higher than the corresponding empirical values, they are considerably lower than what would be implied by the SPIH. As before, this is brought about by very small movements in the interest rate.

Notes

1/L. Hansen (1985) and Sargent (1986) study a general equilibrium, representative agent growth model which rationalizes the SPIH. The essential feature of their model is that the technology shock only affects the average product of capital; the marginal product is constant. Also, utility is quadratic in consumption, and hours are not in the model.

2/If $\theta(L)y_t = C(L)\epsilon_t$ is the ARMA representative for income y_t and r is the real (assumed fixed) rate of interest, then the annuity value of an innovation ϵ_t in y_t is $r C((1+r)^{-1})/\theta((1+r)^{-1})$. (See Deaton, 1986.)

3/One interpretation of the immortal representative agent is Aiyagari's (1986). In a framework that nests mine, he shows how a utility function expressed in terms of per capita consumption and hours, like (1), can summarize preferences in an economy that has a growing number of overlapping generations of people with finite lives and operative bequest motives. Like mine, Aiyagari's is a model with uncertainty in which per capita consumption, capital, and output grow on average.

4/The approximate decision rules for k_t and h_t implied by these values are $k_t = z_t \exp\{9.75 + .9494[\log(k_{t-1}) - \log(z_{t-1}) - 9.75] - .9441(x_t - .00325)\}$ and $h_t = \exp\{5.78 - .4540[\log(k_{t-1}) - \log(z_{t-1}) - 9.75] + .5201(x_t - .00325)\}$. (See Christiano, 1986b, for details.) Decision rules for the model's other variables are derived using (2) and the definition of the production function.

5/I estimate this by dividing the standard deviation of the innovation to consumption by the corresponding quantity for income, as implied by a three-lag vector autoregression in $c_t - c_{t-1}$ and $y_t - y_{t-1}$. My estimate of 33 percent is consistent with Deaton's (1986) estimate of 50 percent since my measure of y_t includes capital income, whereas his only includes labor income. Labor income is a fairly steady 66 percent of GNP. (See Christiano, 1986a, n. 2.1.)

6/For example, while T-bills may be close to risk free, the average household cannot borrow much, if at all, at this rate. In addition, the return on T-bills reflects not just their function of transferring consumption intertemporally, but also their function of providing liquidity. The model abstracts from the latter.

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Table 1--Selected Second-Moment Properties

Statistic ^a	Model Simulations ^b		U.S. Estimates ^c
	Mean	(Standard Deviation)	
σ_c/σ_y	.44	(.031)	.49
σ_{dk}/σ_y	2.56	(.170)	1.91
σ_h/σ_y	378.23	(93.80)	669.59
σ_r/σ_y	.0777	(.0196)	2.24
σ_y	.0176	(.0012)	.0115
$E r_t$.017	(.0009)	.0024
$\rho_{r,\Delta c}(0)$.533	(.031)	.085
$\rho_{\Delta c,\Delta c}(1)$.059	(.108)	.271
$\rho_{\Delta y,\Delta y}(1)$	-.119	(.093)	.361

^a σ , E are the volatility and mean, respectively, of the indicated variable. $\rho_{u,v}(\tau)$ is the correlation between $u(t)$ and $v(t-\tau)$, $\tau=0, 1$. $\Delta u(t)$ denotes $\log u(t) - \log u(t-1)$.

^b1,000, each 112 quarters long.

^c1956.2-1984.1.

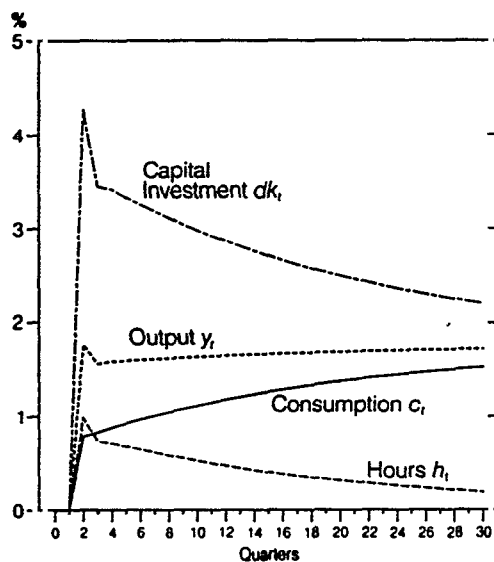


Figure 1. Response of Model to $\epsilon_2 = .019$:
% Deviations From Steady-State Baseline