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Estimating Substitution Elasticities in Household Production Models*

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ABSTRACT

Dynamic general equilibrium models that include explicit household production sectors provide a useful framework within which to analyze a variety of macroeconomic issues. However, some implications of these models depend critically on parameters, including the elasticity of substitution between market and home consumption goods, about which there is little information in the literature. Using the PSID, we estimate these parameters for single males, single females, and married couples. At least for single females and married couples, the results indicate a high enough substitution elasticity that including home production will make a significant difference in applied general equilibrium theory.

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1. Introduction

Recently, dynamic general equilibrium models with explicit household production sectors have been shown to provide a useful framework within which to analyze a variety of macroeconomic issues.¹ Benhabib et al. (1991) and Greenwood and Hercowitz (1991) show how real business cycle models with explicit home production sectors are better able to account qualitatively and quantitatively for several aspects of aggregate economic time series. McGrattan et al. (1993) use a dynamic general equilibrium model with household production to study fiscal policy and find predictions that differ significantly from those implied by models without home production. Rios-Rull (1993) and Braun and McGrattan (1994) use home production models to analyze labor market issues. Baxter and Jermann (1994) use a closely related structure to interpret consumption data, and Canova and Ubide (1994) use a two-country version of the model to study several issues related to international business cycles.

A key way in which all of these models differ from standard dynamic general equilibrium models (that do not explicitly incorporate home production) is that the presence of the household sector allows individuals to substitute along additional margins. For example, in a standard one-sector growth model, at each point in time output is divided between consumption and

¹ By *household production*, we mean activities like cooking, cleaning, child care, and so on, that take place in the home rather than the formal market sector, but are nevertheless analogous to market production in the sense that they use labor and capital as inputs and yield consumption goods as outputs. Note that although explicit modeling of household production is relatively new in macroeconomics, it has been studied by labor economists for some time; see, for example, Becker (1965, 1988) and Gronau (1986). But that research typically does not use dynamic general equilibrium theory.

investment, while in a home production model, output must be divided among consumption, business investment, and household investment. Hence, home production models have richer implications for the capital market. Similarly, because time must be divided among leisure, market work, and home work, rather than simply between leisure and work, home production models have richer implications for the labor market. Depending on individuals' willingness and incentives to substitute between the home and market sectors, these extra margins can have important implications for the effects of things like tax or productivity changes.

Consider first the effect of an increase in the labor income tax rate. In a standard model, the magnitude by which workers change the number of hours they allocate to market work depends on how willing they are to substitute leisure for market consumption goods. In a model with home production, in addition to increasing leisure, individuals can also increase hours in home production. If individuals are relatively willing to substitute home-produced for market-produced consumption, the result is a large change in hours of market work. For example, higher taxes may cause individuals who are working in the market and paying for child care services to stop working in the market and provide their own child care. Of course, this depends on how willing people are to substitute between market- and home-produced child care.

Now consider standard real business cycle models driven by productivity shocks (for example, the base model in Hansen 1985). The extent to which individuals vary their hours of market work in such models depends on their willingness to intertemporally substitute leisure, and in many reasonable specifications of the model, hours of market work are not volatile enough when compared to the data. In a model with home production, individuals can not only substitute leisure at one date for leisure at other dates, they can also

substitute work in the market for work in the home at a given date, which leads to greater fluctuations in hours of market work.

Generally, the extent to which including home production in economic models makes a difference depends critically on elasticities of substitution between market and home consumption goods. As emphasized by Kydland (forthcoming), however, there does not exist a great deal of information regarding these elasticities, and much of the literature that uses dynamic general equilibrium models with home production has had to choose important parameters more or less arbitrarily; see the discussion in Greenwood et al. (forthcoming) for details. The objective of this paper is to provide some more systematic measurement of the willingness of households to substitute between home and market consumption by estimating the relevant elasticities using microeconomic data.²

We begin by specifying dynamic stochastic decision problems explicitly incorporating household production, both for the case of single individuals and for the case of married couples. Optimization imposes restrictions on consumption, market work, home work, and wages, all of which are observable in our data. We focus on a subset of the restrictions that do not depend at all on the functional form of momentary utility, as long as we assume that market- and home-produced consumption goods can be combined according to a constant elasticity of substitution aggregator, as can hours of market and home work.³

² As discussed in detail below, our data are from the Panel Study of Income Dynamics (PSID), which has direct measures of the time spent working in the household. An alternative source for microeconomic evidence on hours of home production is the Michigan Time Use Survey (see Hill 1985 or Juster and Stafford 1991). This survey may provide a more reliable measure of home hours than the PSID, but its data are available only in a few cross sections, taken several years apart. Moreover, the PSID is better for our purposes because it contains information on consumption expenditure.

³ For example, we do not need to make any assumptions regarding the way utility depends on the consumption and hours aggregators. Moreover, the

These rather weak assumptions are enough to deliver log-linear relations among observable variables, which we estimate using instrumental variables procedures.

The coefficients in the linear equations are of interest in their own right: they describe the relationship between hours of home work and hours of market work, wages, and consumption. More importantly, for our purposes, they can be used to identify the underlying structural parameters of the home production technology and preferences. Perhaps the main finding in this regard is that, at least for households consisting of single females or married couples, the elasticity of substitution between market- and home produced consumption goods is fairly high. Our measures of this elasticity are certainly large enough that household production models calibrated to these numbers would give significantly different answers from models without home production regarding questions in business cycle theory or fiscal policy analysis.

The only other research of which we are aware that attempts to estimate this elasticity is McGrattan et al. (1993). That work uses an aggregate model and postwar U.S. time series. The estimates are obtained using data on market variables, because (with the exception of investment in household capital) there do not exist aggregate time series on household inputs or outputs. It seems of interest to also pursue the alternative strategy of using micro data that explicitly include information on time spent in home production. Our findings are largely consistent with those in McGrattan et al. Therefore, at least in this instance, measurement using microeconomic and macroeconomic data

relations we exploit do not depend at all on auxiliary assumptions such as intertemporal separability or stationarity in preferences or on access to credit or insurance markets.

yields similar conclusions.

2. Theoretical Model

We begin with a decision problem that is similar to those solved by households in existing macroeconomic models with home production, in which the household is modeled as if it consists of only one person. Later we extend this to the case of households that consist of more than one person. In both versions, the model is dynamic and stochastic: t indexes the date and s the state of nature. For simplicity, both t and s are discrete.

In the case of a household consisting of a single individual, the commodity space X contains as a typical element a function x that maps (t,s) into a point in \mathbb{R}_+^4 . In particular, for each (t,s) ,

$$(1) \quad x(t,s) = [c_M(t,s), c_H(t,s), h_M(t,s), h_H(t,s)],$$

where $c_M(t,s)$ denotes consumption of a commodity purchased in the market, $c_H(t,s)$ denotes consumption of a commodity produced in the home, $h_M(t,s)$ denotes hours worked in the market, and $h_H(t,s)$ denotes hours worked in the home.⁴ Preferences are described by a utility function $U: X \rightarrow \mathbb{R}$. This is a very general specification; for instance, we do not need to assume that U is additively separable with respect to t or even with respect to s (that is, we do not even need to assume von Neuman-Morgenstern utility), although the

⁴ This is one version of the standard household production model used in labor economics; see Gronau (1986), Section V. Another version, which is the one discussed by Becker (1965), does not include the hours variables directly in the utility function (although they enter indirectly through the constraints). Sometimes there are good theoretical reasons for including hours directly; see, for example, the application in Nosal et al. (1992).

special case usually considered in existing work assumes separability with a stationary momentary utility function and a constant discount rate.

The individual seeks to maximize U subject to several constraints.

First, there is the standard budget constraint

$$(2) \quad \sum_{t,s} q(t,s)[c_M(t,s) - w(t,s)h_M(t,s)] \leq I,$$

where I is exogenous nonlabor income, $q(t,s)$ is the price of the market consumption good, and $w(t,s)$ is the real wage (i.e., the price of labor in terms of the consumption good) at (t,s) . We write constraint (2) assuming complete contingent commodity markets; however, the estimating equations we derive below also hold in economies with borrowing constraints or other incomplete-market complications. This is because these equations follow exclusively from the optimization conditions at every date-state pair, which hold even if the household is constrained in credit or insurance markets.

Second, there is the home production constraint:

$$(3) \quad c_H(t,s) \leq g[h_H(t,s); t,s] \text{ for all } (t,s).$$

The defining characteristic of home-produced goods is that they must be produced by the household that consumes them. Note that we do not include capital in the home production function (since the data we use below unfortunately do not contain information on household capital), although we do allow the function g to depend on both the date and state. Finally, there is the constraint that says market plus home work cannot exceed some fixed number of total hours \bar{H} ; we write this as $L(t,s) \geq 0$ for all (t,s) , with the interpretation of $L(t,s) = \bar{H} - h_M(t,s) - h_H(t,s)$ as leisure.

We assume that the household's utility function U is twice continuously differentiable, strictly increasing in the two consumption variables, and strictly decreasing in the two hours variables. Then, as long as we assume an interior solution (which we always do for single individuals), we have the following first order conditions: for all (t,s) ,

$$(4) \quad \partial U / \partial c_M(t,s) - \lambda_M q(t,s) = 0$$

$$(5) \quad \partial U / \partial h_M(t,s) + \lambda_M q(t,s)w(t,s) = 0$$

$$(6) \quad \partial U / \partial c_H(t,s) - \lambda_H(t,s) = 0$$

$$(7) \quad \partial U / \partial h_H(t,s) + \lambda_H(t,s)g'[h_H(t,s);t,s] = 0,$$

where λ_M is the multiplier on the (single) market budget constraint and $\lambda_H(t,s)$ is the multiplier on the home production constraint at (t,s) .⁵

Equations (4) and (5) are the standard first order conditions studied in the empirical labor supply literature, and (6) and (7) are the analogs that emerge from introducing home production. For example, MaCurdy (1981) uses (5) to derive restrictions on market hours and wages at different dates while Altonji (1986) combines (4) and (5) to derive restrictions on market hours, wages, and consumption at each date. These authors are primarily interested in intertemporal labor supply elasticities. Since our interest is the

⁵ As is standard, the second order conditions necessarily hold if U is quasi-concave. However, we find some evidence in the empirical work below that it is not and, in particular, that individuals may prefer to specialize in work in the home or work in the market, rather than a convex combination. But since the home production constraint is nonlinear, an interior maximum can still obtain even if U is not quasi-concave.

elasticity of substitution between c_M and c_H , we are able to impose much weaker restrictions on the utility function in what follows.

Elimination of the multipliers from (4)-(7) yields the following conditions: for all (t,s) ,

$$(8) \quad w(t,s)\partial U/\partial c_M(t,s) = -\partial U/\partial h_M(t,s)$$

$$(9) \quad g'[h_H(t,s);t,s]\partial U/\partial c_H(t,s) = -\partial U/\partial h_H(t,s).$$

Then (8) and (9) combine to yield the following relationship: for all (t,s) ,

$$(10) \quad \frac{w(t,s)}{g'[h_H(t,s);t,s]} = \frac{\partial U/\partial c_H(t,s)}{\partial U/\partial h_H(t,s)} \frac{\partial U/\partial h_M(t,s)}{\partial U/\partial c_M(t,s)}.$$

We emphasize that (10) does not depend on auxiliary assumptions on the utility function, like separability or stationarity. Nor does it depend at all on complete insurance or credit markets, and exactly the same relation can be derived without these assumptions.

Of course, one has to make *some* parametric assumptions in order to have parameters to estimate. Since we are interested in the elasticities of substitution between home and market variables, we assume utility can be written in terms of constant elasticity aggregators of the two consumptions and the two hours.

Assumption 1: For all (t,s) , U depends on $c_M(t,s)$ and $c_H(t,s)$ only through the aggregator

$$(11) \quad C(t,s) = \left[a_M c_M(t,s)^\theta + a_H c_H(t,s)^\theta \right]^{1/\theta}$$

and depends on $h_M(t,s)$ and $h_H(t,s)$ only through the aggregator

$$(12) \quad H(t,s) = \left[b_M h_M(t,s)^\gamma + b_H h_H(t,s)^\gamma \right]^{1/\gamma}.$$

We also specify the home production function as follows:

Assumption 2: $g(h_H; t, s) = B(t, s) h_H^\eta$.

It is the parameters θ , γ , and η that are of interest.

Assumption 1 implies that equation (10) reduces to

$$(13) \quad \frac{w(t,s)}{g'[h_H(t,s); t, s]} = \frac{a_H b_M}{a_M b_H} \left[\frac{c_H(t,s)}{c_M(t,s)} \right]^{\theta-1} \left[\frac{h_M(t,s)}{h_H(t,s)} \right]^{\gamma-1}.$$

Inserting g' from Assumption 2, taking logs, and simplifying, we arrive at

$$(14) \quad \log(h_H) = \alpha_0 + \alpha_1 \log(w) + \alpha_2 \log(c_M) + \alpha_3 \log(h_M),$$

where we have suppressed (t,s) indices and the constants satisfy

$$\alpha_0 = \log(a_M b_H / \eta a_H b_M) / (\eta\theta - \gamma) - \log[B(t,s)]\theta / (\eta\theta - \gamma)$$

$$\alpha_1 = 1 / (\eta\theta - \gamma), \quad \alpha_2 = (\theta - 1) / (\eta\theta - \gamma), \quad \alpha_3 = (1 - \gamma) / (\eta\theta - \gamma).$$

In our empirical work we assume that η , γ , and θ are the same for all individuals (except that we allow them to differ between men and women). The parameters a_M , a_H , b_M , b_H , and $B(t,s)$ may differ across individuals.

Different assumptions on how these parameters vary across individuals give rise to different estimation strategies, as we will discuss in more detail below.

Up to this point, we have explicitly considered households consisting of single individuals. Theoretically, it is often useful to abstract from interpersonal relationships within the family and model the household as a single decision-making entity. This can be problematic, however, when using the model to interpret data on families consisting of two or more people, such as a husband and a wife. One reason is that the data typically indicate that the husband and the wife face different wage rates and supply different amounts of labor. One could aggregate the data in some ad hoc manner--say, one could use the average wage and the average hours of the couple--but this is a waste of useful information. Hence, we now develop a version of the theory for two-person households (the results easily generalize to N person households).

Consider a utility function U defined on X^2 , the commodity space that takes as elements $[x_1(t,s), x_2(t,s)]$, where $x_i(t,s)$ is the vector of market and home consumption and market and home work at (t,s) for individual i . It is possible to interpret the function U as family utility; alternatively, it is possible to interpret U as a Nash product, the maximization of which yields the solution to an intrafamily bargaining problem.⁶ In any case, the

⁶ Let $U_i(x_i)$ be the utility and \bar{U}_i be the threat point of individual i . If

$$U(x_1, x_2) = [U_1(x_1) - \bar{U}_1][U_2(x_2) - \bar{U}_2],$$

then the Nash bargaining solution maximizes U (see, for example, Osborne and Rubinstein 1990). Hence, we can interpret the household objective function as a reduced form for an intrafamily bargaining problem. See Lundberg and Pollak (1993) for a discussion of alternative models of bargaining within the family.

household is subject to the budget and home production constraints:

$$(15) \quad \sum_{t,s} q(t,s) \sum_i [c_{Mi}(t,s) - w_i(t,s)h_{Mi}(t,s)] \leq I$$

$$(16) \quad \sum_i c_{Hi}(t,s) \leq g[h_{H1}(t,s), h_{H2}(t,s); t, s] \text{ for all } (t, s).$$

where the wage rate, as well as consumption and hours in both the home and the market, are indexed by family member.

From the first order conditions one can derive a version of (10) and, under Assumptions 1 and 2, a version of (14) for each individual. However, the regression equation for each i then has consumption for each i on the right-hand side, whereas the available data have market consumption only for the household and not for each individual in the household. Therefore, we take an alternative route and assume that the family utility function is defined over a commodity space Y that contains as a typical element a function y that maps each date and state into a point in \mathbb{R}_+^6 : for each (t, s) ,

$$(17) \quad y(t, s) = [c_M(t, s), c_H(t, s), h_{M1}(t, s), h_{M2}(t, s), h_{H1}(t, s), h_{H2}(t, s)],$$

where $c_M(t, s) = \sum_i c_{Mi}(t, s)$ and $c_H(t, s) = \sum_i c_{Hi}(t, s)$ denote the total family consumption of the two commodities.

For this household, we generalize Assumption 1 as follows:

Assumption 1': For all (t, s) , U depends on $c_M(t, s)$ and $c_H(t, s)$ only through the aggregator

$$(18) \quad C(t,s) = \left[a_{MM} c_M(t,s)^\theta + a_{HH} c_H(t,s)^\theta \right]^{1/\theta},$$

and, for each i , U depends on $h_{Mi}(t,s)$ and $h_{Hi}(t,s)$ only through the aggregator

$$(19) \quad H_i(t,s) = \left[b_{Mi} h_{Mi}(t,s)^{\gamma_i} + b_{Hi} h_{Hi}(t,s)^{\gamma_i} \right]^{1/\gamma_i}.$$

We also generalize Assumption 2 as follows:

$$\text{Assumption 2': } g(h_{H1}, h_{H2}; t, s) = B(t, s) h_{H1}^{\eta_1} h_{H2}^{\eta_2}.$$

More general specifications for a two-person home production function could be imagined; e.g., it may be of interest to estimate the elasticity of substitution between the two labor inputs. We adopt Assumption 2' here for simplicity and leave such generalizations to future work.

Following the procedure used for single individuals, one can show that interior solutions imply that

$$(20) \quad \log(h_{H1}) = \alpha_{10} + \alpha_{11} \log(w_1) + \alpha_{12} \log(c_M) + \alpha_{13} \log(h_{M1}) + \alpha_{14} \log(h_{H2})$$

$$(21) \quad \log(h_{H2}) = \alpha_{20} + \alpha_{21} \log(w_2) + \alpha_{22} \log(c_M) + \alpha_{23} \log(h_{M2}) + \alpha_{24} \log(h_{H1})$$

where the constants α_{ij} are functions of the parameters θ , γ_1 , γ_2 , η_1 , and η_2 . Equations (20) and (21) generalize (14) from the model with a household consisting of a single individual; the main difference is that home hours of one individual enter the equation for the other, since the two individuals interact in the household production function.

One can then solve (20) and (21) for $\log(h_{H1})$ and $\log(h_{H2})$:

$$(22) \quad \log(h_{H1}) = \beta_{10} + \beta_{11} \log(w_1) + \beta_{12} \log(w_2) + \beta_{13} \log(c_M) \\ + \beta_{14} \log(h_{M1}) + \beta_{15} \log(h_{M2}),$$

$$(23) \quad \log(h_{H2}) = \beta_{20} + \beta_{21} \log(w_1) + \beta_{22} \log(w_2) + \beta_{23} \log(c_M) \\ + \beta_{24} \log(h_{M1}) + \beta_{25} \log(h_{M2}).$$

The constants β_{10} depend on s and t and can be randomly distributed across households, while the other coefficients are given by

$$\begin{aligned} \beta_{11} &= \Delta(\eta_2^{\theta-\gamma_2}) & \beta_{21} &= -\Delta\theta\eta_1 \\ \beta_{12} &= -\Delta\theta\eta_2 & \beta_{22} &= \Delta(\eta_1^{\theta-\gamma_1}) \\ \beta_{13} &= -\Delta\gamma_2(\theta-1) & \beta_{23} &= -\Delta\gamma_1(\theta-1) \\ \beta_{14} &= \Delta(1-\gamma_1)(\eta_2^{\theta-\gamma_2}) & \beta_{24} &= -\Delta\theta\eta_1(1-\gamma_1) \\ \beta_{15} &= -\Delta\theta\eta_2(1-\gamma_2) & \beta_{25} &= \Delta(1-\gamma_2)(\eta_1^{\theta-\gamma_1}) \end{aligned}$$

where $\Delta \equiv 1/(\gamma_1\gamma_2^{-\theta}\eta_1\gamma_2^{-\theta}\eta_2\gamma_1)$. Note that we have more equations than parameters; in fact, we can solve for θ , η_1 , η_2 , γ_1 , and γ_2 from either of the two equations independently. Hence, system (22)-(23) entails cross-equation restrictions which can be tested (see below).

As indicated above, this procedure is valid if we have an interior solution. In our data, however, some married couples have zero market hours

in some periods for the wife. While there are several ways of dealing with this issue, as discussed in the next section, for now we simply observe that equation (20) holds for the husband even if the wife supplies zero market hours (assuming that individual 1 is the husband and individual 2 is the wife). One could estimate equation (20) instead of system (22)-(23) and still identify some of the economic parameters of interest. In particular, the coefficients of (20) can be solved for θ , η_1 , η_2 , and γ_1 , but not for γ_2 .

3. Empirical Results

We use data from the Panel Study of Income Dynamics (PSID), for the years 1976-1987. We partition the sample into three groups: single males, single females, and married couples. For single men and women, the sample includes all individuals between the ages of 25 and 66 who had at least 30 usual hours of market work per week and worked at least 40 weeks during the year. The reason for excluding single people who are not working full time is that we do not model income support programs and their interaction with labor supply.

For married individuals the sample includes all individuals between the ages of 17 and 72 who are part of a couple in which the husband had at least 30 usual hours of market work per week and worked at least 40 weeks per year. There was no selection criterion for the hours of women in the sample of married couples. The final sample includes 627 observations for single men, 845 for single women, and 4839 for married couples. When we estimate system (22)-(23), which is derived assuming interior solutions, we throw out observations for which market hours are zero for the wife; we will discuss this further below.

The PSID asks individuals how many hours of market and home work they carry out in a usual week. These are the two measures of hours that we use in our empirical work. Table 1 displays average hours of market and home work for single and married men and women. The numbers indicate that the average female works less than the average male in the market, but more in the home, and that the average married female works more in the home than the market.⁷ Market wages correspond to average hourly earnings deflated by the consumer price index. The only measure of consumption in the PSID is household food consumption, and we use this as a proxy for c_M . We include expenditures for food consumed at home as well as expenditures on restaurant meals.

Before proceeding with the estimation results, several issues require some discussion. First, the specification of the error terms, α_0 in (14) and β_{10} and β_{20} in (22)-(23), will influence the choice of estimation technique. For example, if one assumes that the parameters a_H , a_M , b_H , b_M , and B are randomly distributed across households at each point in time but are constant across time for a given household, then the estimating equations contain individual fixed effects which must be addressed--say, by transforming the data before estimation by taking deviations from individual means.

⁷ Although the general pattern is similar, the exact numbers are somewhat different from those implied by the Michigan Time Use Survey, which obtains data from individuals who fill out time diaries. For example, Table 1 indicates that an average family in our sample spends 81.8 hours in market work and 24.5 hours in home work per week. The time use numbers from 1981 indicate that an average family spends 73.4 hours in market work and 44.3 hours in home work, and the time use numbers from 1965 indicate that an average family spends 76.9 hours in market work and 53.3 hours in home work (computed from Juster and Stafford 1991, Table 3). The main difference from our perspective is that the time use data indicate far more time spent in home work. This is at least partly due to the fact that our selection criterion rules out households in which no one works in the market, which are the households with the most home production.

Alternatively, if one assumes that these parameters are randomly distributed across the population and are i.i.d. across periods for a given household, then it is appropriate to estimate the equations in levels.

An elaboration on these two cases is to assume that a_H , a_M , b_H , b_M , and B also contain a component which depends systematically on demographic variables. Of particular interest here is the possibility that the presence of young children may influence the parameters. In what follows, we report results for the case in which we estimate using levels of the relevant variables and include a dummy variable for the presence of children under six. We also carried out the estimation using differences from means, but found that the results did not differ markedly; hence, these results are not reported.

A second issue concerns the fact that the right-hand side variables are likely to be correlated with the error term (e.g., the value of B affects the error term and the choice of market hours). We deal with this by using an instrumental variables procedure. In the results reported below, the instruments include the following: For single individuals, we use age, age squared, lagged consumption, and dummies for whether living in a SMSA, whether covered by a union, and whether living in the South. For married couples, to estimate system (22)-(23) or equation (20), we use the husband's age and age squared, the wife's age and age squared, the wife's education, lagged consumption, lagged wife's home work, and the dummies described above. The legitimacy of the instruments was verified at standard significance levels using a Hausman test.

Other issues arise in the context of estimating the system (22)-(23). As discussed previously, these equations were derived assuming an interior solution for both market and home hours of work for both individuals, but in

the data there are many observations in which the wife has zero hours of market work. There are three ways in which one could deal with this situation. The first would be to explicitly incorporate the potential boundary solution and then use the method of maximum likelihood to obtain parameter estimates using the entire sample. Second, one can restrict attention to that part of the sample which does have positive hours of market work. This potentially causes a sample selection bias in the resulting estimates, although previous work has typically not found this particular selection bias to be significant (see, e.g., Heckman and MaCurdy 1980).⁸ Third, as noted earlier, equation (20) holds for husbands as long as they have positive hours of market and home work, even if the wife has zero market hours. Estimating equation (20) involves no selection bias, although it has the disadvantage of not using all of the available data. In our empirical work here, we carry out the latter two procedures and leave the more ambitious task of maximum likelihood estimation for future work.

The linear equations (14) and (20) are estimated using instrumental variables. One can do the same for system (22)-(23) if the overidentifying restrictions are not taken into account. If we impose the cross-equation restrictions, however, the system is nonlinear: hence, we use a method of moments estimator. To construct the standard errors for the structural parameters, we proceed as follows. Consider, for example, θ , which is a nonlinear function of the vector of estimated coefficients--say $\theta = f(\mathbf{b})$. Let \mathbf{d} be the vector of partial derivatives of f with respect to \mathbf{b} . The asymptotic variance-covariance matrix of θ is given by \mathbf{dVd}' , where \mathbf{V} is the

⁸ In a richer model with both intensive and extensive margins (hours per week and weeks per year), another potentially important boundary condition is that some people work all 52 weeks. See Rogerson and Rupert (1991).

variance-covariance matrix of the coefficient estimates.

To begin the empirical analysis, we first estimate equation (14) for single individuals and equation (20) and system (22)-(23) for married couples without imposing any cross-equation restrictions in the latter case. The results are reported in Table 2 (standard errors are in parentheses) and seem very reasonable.⁹ The child dummies are significant and take the expected sign. Home work for both single men and single women is significantly negatively related to their wages. For single men, home work is positively related to market work, but not significantly so, whereas for single women, home work is negatively related to market work and the coefficient is highly significant.

For married men, their home work is negatively related to their wages, although whether the effect is significant depends on whether we look at system (22)-(23) or equation (20). Also, the home work of married men is significantly negatively related to their wives' wages. For married women, home work goes up with their husbands' wages and with their own wages (but the latter effect is not significant). Home work for married men increases with their wives' market work and decreases with their own market work, although whether or not the effect is significant again depends on whether we look at (22)-(23) or (20). Home work for married women decreases with both their own market work and their husbands' market work. Finally, the results for equation (20) indicate that home work for married men increases with their wives' home work; this is consistent with the notion that the marginal product of home hours for one individual increases with the home hours of the other

⁹ Although not reported in the table, for most of the cases R^2 was around 0.1 or slightly higher, which is fairly typical in micro studies of labor supply.

individual.

We now turn to the underlying structural parameters: θ , γ , and η for single individuals and γ_1 , γ_2 , η_1 , and η_2 for married couples. The elasticity of substitution between market- and home-produced consumption is easily computed as $\varepsilon = 1/(1-\theta)$. The case $\theta = 0$ (or $\varepsilon = 1$) means that c_M and c_H are aggregated by a Cobb-Douglas function. As discussed in Benhabib et al. (1991) and Greenwood et al. (forthcoming), this is an interesting benchmark because $\theta = 0$ implies that the introduction of home production into an otherwise standard model does not make a difference, while larger values of θ imply that introducing home production does make a difference, for a variety of economic issues.

The structural parameters are computed as nonlinear functions of the coefficients in the above linear equations. However, recall that system (22)-(23) is overidentified. We tested the cross-equation restrictions and the model passed easily. Hence, we only report parameter estimates for the case where these restrictions are imposed.¹⁰ The results are in Table 3.

For single men, the point estimate of θ is -0.065, implying an elasticity very near unity (although the standard error is large). For single women, the point estimate is $\theta = 0.445$ ($\varepsilon = 1.8$) and is significant. For married couples, the point estimate of θ depends on whether we use the system (22)-(23) or equation (20), but the standard errors are large in both cases. Estimates of the γ 's vary; however, except for single females, they are typically not significantly different from 1 (which is the case of perfect substitution between home and market work). The point estimates of the η 's

¹⁰ Since θ is the parameter in which we are primarily interested, we note for the record that the values of θ implied by (22) and (23) without the cross-equation restrictions are 0.526 and 0.769, with standard errors of 0.151 and 0.348, respectively.

are either implausible (greater than 1 or negative) or have very large standard errors.

Overall, the results in Table 3 are not particularly informative. One interpretation is that it is difficult to estimate all of the parameters of both the home production technology and preferences with the available data. One response is to set the technology parameters (the η 's) exogenously and estimate only the preference parameters (θ and the γ 's). In order to determine appropriate values at which to set the η 's, we consider two pieces of information. First, using maximum likelihood techniques, McGrattan et al. (1993) estimate a macro model with home production from time series data. The estimates imply that $\eta = 0.79$, with a small standard error of 0.081.

Second, Greenwood et al. (forthcoming) calibrate factor shares in the home production function to match capital and labor inputs observed in the National Income and Product Accounts and the Michigan Time-Use Survey data. The result is $\eta = 0.68$. We performed a similar calculation using the PSID rather than the Time-Use Survey data and got a similar number. Based on the estimates in McGrattan et al. and on these numbers, we think that a value for η of around $3/4$ is reasonable for a single-person household. However, we experimented by varying η over a fairly wide range, and the results were virtually unchanged. For households consisting of married couples, a slightly generalized version of the calibration procedure yields approximately $\eta_1 = 0.10$ for men and $\eta_2 = 0.65$ for women.¹¹ We used these numbers as a benchmark,

¹¹ In the two-person household case, we assume home and market work are perfect substitutes for both men and women, a Cobb-Douglas market production function in market capital and efficiency units of market labor (which is defined as a weighted sum of male and female market hours in order to capture differences in wages in the data), and a Cobb-Douglas home production function in home capital, male home hours, and female home hours. Then we follow the procedure in Greenwood et al. (forthcoming).

but also experimented with other settings, some of which are mentioned below.

Table 4 reports the results with η set exogenously. For single men and women, restricting $\eta = 0.75$ yields very similar estimates of θ to the unrestricted case (and, again, this is actually true when we restrict η to any value in a fairly wide range). For married couples, according to system (22)-(23), restricting the η 's increases θ from 0.083 to 0.363, and according to equation (20), restricting the η 's increases θ from 0.355 to 0.750. And notice that in both cases the standard errors go way down. Alternative restrictions deliver similar results. For example, when we set $\eta_1 = \eta_2 = 0.375$, system (22)-(23) yields $\theta = 0.456$ and equation (20) yields $\theta = 0.684$ (and the standard errors in this case are very similar to the case where $\eta_1 = 0.1$ and $\eta_2 = 0.65$).¹²

The estimates of γ_1 and γ_2 in Table 4 typically exceed unity, which violates quasi-concavity of the utility function. That is, other things being equal, individuals prefer specializing in home work or specializing in market work rather than a convex combination. Notice carefully, however, that even if the γ 's exceed 1, this does not necessarily imply that the second order conditions for an interior maximum are violated, because in general the home production function is nonlinear and the two types of consumption goods are not perfect substitutes.¹³

¹² There are some values of the η 's that generate markedly different estimates of θ , but they are values that are difficult to reconcile with what we know about relative wages and the allocation of time. According to our calibration procedure, the observed wage ratio and the ratios of home to market work for men and women imply that η_2 must be bigger than η_1 .

¹³ In any case, we also estimated θ restricting the γ 's to equal 1. The result from system (22)-(23) is that $\theta = 0.213$ with a standard error of 0.032, and the result from equation (20) is that $\theta = 0.338$ with a standard error of 0.006.

4. Conclusion

Although the estimates vary somewhat depending upon the restrictions imposed, the message we take away from the above results is that at least for single females and married couples, there is evidence of a fairly high willingness to substitute between home and market consumption. To put these results into perspective, we return to the use of home production models in applied general equilibrium theory. The computational experiments in Benhabib et al. (1991) or Greenwood et al. (forthcoming) indicate that the behavior of household production models with $\theta = 0$ is very similar to that of models without home production. The values of θ we estimate are sufficiently greater than 0, implying that the presence of home production will have significant effects in these models.

Finally, we compare our results to those based on aggregate time series in McGrattan et al. (1993). That procedure yields a point estimate of $\theta = 0.385$. Hence, the aggregate data generates a result that is consistent with our findings based on microeconomic evidence. While there is clearly need for more work, we conclude that progress is being made in measurement of the household sector, and the results indicate that households are fairly willing to substitute between market- and home-produced goods. Therefore, explicit modeling of the household sector is likely to be an important feature in applied general equilibrium analyses.

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| | <u>Married</u> | | <u>Single</u> | |
|-------------|----------------|--------|---------------|--------|
| | Male | Female | Male | Female |
| Market Work | 43.1 | 20.5 | 42.3 | 39.5 |
| Home Work | 7.2 | 25.7 | 9.6 | 14.9 |

Table 1: Hours of Work per Week

| | <u>Singles</u> | | <u>Married Couples</u> | | |
|----------|-------------------|------------------|------------------------|-------------------|-------------------|
| | (14) | | (22)-(23) | | (20) |
| | h_{H1} | h_{H2} | h_{H1} | h_{H2} | h_{H1} |
| Child | --- | --- | 0.346 (0.065) | 0.435 (0.071) | 0.076 (0.026) |
| w_1 | -0.941 (0.227) | --- | -0.014 (0.147) | 0.294 (0.105) | -0.114 (0.029) |
| w_2 | --- | -1.97 (0.306) | -0.144 (0.069) | 0.055 (0.049) | --- |
| c_M | 1.00 (0.188) | 1.09 (0.167) | 0.036 (0.106) | -0.165 (0.076) | 0.073 (0.030) |
| h_{M1} | 0.611 (1.14) | --- | -1.94 (0.778) | -1.22 (0.555) | -0.023 (0.084) |
| h_{M2} | --- | -5.35 (0.902) | 0.314 (0.138) | -0.292 (0.099) | --- |
| h_{H2} | --- | --- | --- | --- | 0.197 (0.020) |

Table 2: Estimation Results

| | θ | γ | | η | |
|-----------------------|-------------------|------------------|------------------|-------------------|-------------------|
| | | <u>Male</u> | <u>Female</u> | <u>Male</u> | <u>Female</u> |
| Single Males | -0.065 (0.471) | 1.65 (1.24) | --- | 1.67 (5.63) | --- |
| Single Females | 0.445 (0.121) | --- | -1.71 (0.399) | --- | -0.597 (0.037) |
| Couples: (22)-(23) | 0.083 (0.292) | 2.97 (1.59) | 0.980 (0.052) | -150.0 (591.0) | 4.08 (28.7) |
| Couples: (20) | 0.355 (0.273) | 0.800 (0.882) | --- | -22.5 (20.2) | 4.86 (15.1) |

Note: elasticity of substitution between c_M and c_H is $\varepsilon = \frac{1}{1-\theta}$

Table 3: Parameter Estimates

| | θ | γ | | η | |
|-----------------------|-------------------|-----------------|-----------------|-------------|---------------|
| | | <u>Male</u> | <u>Female</u> | <u>Male</u> | <u>Female</u> |
| Single Males | -0.076 (0.432) | 1.47 (0.142) | --- | 0.75 | --- |
| Single Females | 0.506 (0.146) | --- | 1.25 (0.062) | --- | 0.75 |
| Couples: (22)-(23) | 0.363 (0.069) | 5.79 (2.48) | 2.66 (0.030) | 0.1 | 0.65 |
| Couples: (20) | 0.750 (0.123) | 4.14 (0.532) | --- | 0.1 | 0.65 |

Note: elasticity of substitution between c_M and c_H is $\varepsilon = \frac{1}{1-\theta}$

Table 4: Parameter Estimates with η Fixed