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Optimal Regulation in the Presence of Reputation Concerns*

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ABSTRACT

In all markets, firms go through a process of creative destruction: entry, random growth and exit. In many of these markets there are also regulations that restrict entry, possibly distorting this process. We study the public interest rationale for entry taxes in a general equilibrium model with free entry and exit of firms in which firm dynamics are driven by reputation concerns. In our model firms can produce high-quality output by making a costly but efficient initial unobservable investment. If buyers never learn about this investment, an extreme "lemons problem" develops, no firm invests, and the market shuts down. Learning introduces reputation incentives such that a fraction of entrants do invest. We show that, if the market operates with spot prices, entry taxes always enhance the role of reputation to induce investment, improving welfare despite the impact of these taxes on equilibrium prices and total production.

Keywords: Creative destruction; Reputation; Regulation; Entry and exit; Firm dynamics; General equilibrium
JEL: D21, D82, L15, L51

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I. Introduction

Creative destruction in capitalist economies is ubiquitous: even within narrowly defined industries, there is an ongoing process of churning through which firms enter, undergo random growth, and exit. At the same time, attempts by governments to regulate this process of creative destruction with entry barriers are also ubiquitous, as documented by Djankov, La Porta, Lopez-de Silanes, and Shleifer (2002). Is there a public interest rationale for this regulation of entry? And if so, what are the foundations of this rationale?

One leading view is that firm dynamics are driven by firms' heterogeneous productivities and that attempts to regulate the process of firm entry, growth, and exit are important impediments to aggregate productivity that reduce welfare.[1] An alternative view is that firm dynamics are driven by uncertainty about the *quality* of new products or services. If it takes buyers time to learn about the quality of entering firms, these firms initially face lower demand and prices until they are able to establish a good reputation for their product. In this view, firms enter small, and their growth and exit are driven by the evolution of their reputation in the market.[2]

Concerns about quality are indeed critical in many markets. Firms often make upfront investments in the quality and safety of their product. Buyers, in turn, interact infrequently with these firms through spot markets, and it may be difficult to enforce appropriate and timely compensation for the damages that poor quality products may generate in terms of health, safety hazards, or opportunity costs. Examples range widely from food or drug safety to many durable goods, and to professional and consumer services such as doctors, lawyers, home contractors, hotels, restaurants, etc.[3] In all these markets, quality concerns limit the buyers' willingness to pay, and

[1] Hopenhayn (1992), Melitz (2003), and Luttmer (2007) present models of the process of creative destruction driven by the dynamics of firm productivities. Herrendorf and Teixeira (2011) examine the welfare costs of entry regulation in such a model. Hopenhayn and Rogerson (1993), Parente and Prescott (1999), Hsieh and Klenow (2009), Restuccia and Rogerson (2008), and Fattal Jaef (2013) consider other interventions into this process of creative destruction.

[2] Foster, Haltiwanger, and Syverson (2012) present evidence that demand rather than supply factors drive firm and plant dynamics. Klein and Leffler (1981), Bar-Isaac (2003), Board and Meyer-ter-Vehn (2013 and 2012), and Vial and Zurita (2012) present reputation-driven models of firm dynamics.

[3] Even if firms voluntarily guarantee their products against defects, it is still costly for buyers to use warranties (it is not always clear which defects they cover, it takes time and resources to replace damaged goods, etc) and they prefer to pay higher prices for products from reputable firms rather than buy from firms with worse reputations and risk having to use warranties.

this reduces the firms' returns to making those upfront quality investments.

Guaranteeing a minimum product quality is usually cited as the main Pigouvian or "public interest" rationale for regulating firm entry. Regulatory entry barriers are meant to discourage firms from entering with low quality products.[4] At the same time, however, regulatory entry barriers impose additional costs which firms pass on to consumers through higher prices. Moreover, regulatory interventions may not be needed to enhance quality if the market eventually recognizes quality and rewards firms through reputational mechanisms. This suggests that regulation leads to a non-trivial tradeoff between enhancing quality and distorting prices, with ambiguous welfare consequences.

In this paper, we analyze the welfare implications of regulatory intervention into the process of firm entry, growth, and exit through the lens of a simple general equilibrium model of firm dynamics driven by reputational considerations. As a general matter, we show that entry regulation is welfare-enhancing, even though reputational incentives already mitigate adverse selection in the unregulated market. In fact, we argue that appropriate regulatory interventions complement and enhance the reputational incentives that govern firm dynamics.

In the model, a firm produces high quality output only if it makes a costly initial investment in quality upon entering the market. Firms can also enter the market without making this initial investment, but the shoddy output of these low quality firms detracts from, rather than adds to, social welfare. In equilibrium, firms invest in quality only if they expect to recoup the investment costs through subsequent quasi-rents from selling their goods to buyers at a price above marginal cost.

If buyers perfectly observe the firms' investment in quality, then the spot market equilibrium is fully efficient, as in Hopenhayn (1992). Instead, we assume that these investments are not observable by outsiders, and information about entering firms diffuses only gradually through a process of public signals about the firms' quality.[5] Some low-quality firms will then enter

[4]See, for example, Pigou (1938), Klein and Leffler (1981) and Shapiro (1983 and 1986).

[5]Learning about a firm's quality could be the result of public ratings such as those provided by Consumer Reports (for many durable goods), the Better Business Bureau (for a wide range of businesses), US News and World Report (for colleges), Martindale Hubble (for lawyers), Healthgrades (for doctors), Angie's List (for many

the market to extract information rents by pooling at least temporarily with their high-quality peers. This pooling reduces the quasi-rents high quality firms can extract, which lowers their investment incentives. The laissez-faire equilibrium thus features strictly lower average quality, a smaller overall production level, and lower welfare than the first-best allocation with perfect information.

We develop two principal results on the welfare enhancing role of regulation. First, we show that a simple combination of entry fees and price subsidies can implement an allocation arbitrarily close to the first-best with perfect information, even without altering the buyers' information about product quality. A regulator can use these two instruments to target both quality and production, eliminating entry of low quality firms almost completely, while encouraging efficient entry of high quality firms.

Second, we show that small positive entry costs are welfare-improving even if they are the only regulatory tool available. The positive impact of small entry fees on quality more than outweighs the potential negative impact of these fees on equilibrium prices and total production. Indeed, for a wide range of information structures, small entry fees result not only on higher average quality but also in *lower* reputation-adjusted equilibrium prices, *more* aggregate production and higher welfare.

We derive these two main results by analyzing the equilibrium condition that both low and high quality firms earn zero profits at entry. In equilibrium, the quasi-rents earned by a firm of either quality depend on its current reputation and the overall level of production and prices. At the laissez-faire equilibrium in our model, free entry drives the reputation of entering firms to a level that completely dissipates the rents for low-quality entrants. The overall production and price level adjust to offer high quality entrants just enough quasi-rents to compensate for the investment cost.

household services), AAA and Trip Advisor (for tourist hotels), and Zagats and Michelin (for restaurants), for example. Alternatively, one might interpret learning as the noisy observation of previous buyers' experience with the firm's output. This learning can take a very public, and sometimes dramatic, form with cases of product failures such as Bridgestone/Firestone tires recalls after several accidents in 2000, Mattel recalls for lead painted toys in 2007, Toyota recalls for unintended car accelerations in 2009, or the European horse meat scandal in 2013.

A regulatory entry fee limits rent dissipation for low-quality entrants, resulting in relatively fewer low quality entrants and a higher entry reputation than at the laissez-faire equilibrium. This confirms the Pigouvian argument that entry fees indeed enhance quality. But the fee also affects overall production and prices through the incentives to invest in quality. To prove our first result, we show that with a price subsidy, the regulator can correct the effect of entry fees on prices and align the private and social returns for high quality entrants, while using the entry fee to almost completely eliminate entry by low quality firms.[6]

Even if entry fees are the only regulatory instrument, imposing small entry fees is still welfare improving, because the distortions in production only have second-order welfare costs relative to the gains from enhanced quality. In fact, for a broad range of information structures that we analyze, the high quality firms' quasi-rents increase more than one-for-one with small entry fees, so imposing such a fee enhances these firms' ability to recoup the costs of their investment in quality, resulting in more entry by high quality firms *and* more total production. Once entry fees become large enough, however, further increases start to lower entry incentives by high quality firms, and thus reduce aggregate production. The optimal entry fee is therefore positive, but finite.

Our model is cast in discrete time, and in its simplest version the market observes a single signal of firm quality at the end of the period during which a firm entered. No further signals are realized for incumbents, and hence their reputations are no longer updated. This assumption may not seem particularly realistic, but it allows us to present our core results in the simplest possible setting. This signal structure is also a natural extension of the information structure in Klein and Leffler (1981).

In a second version, we allow for ongoing arrival of new signals, which seems more realistic but is also more complicated to analyze. To keep the analysis tractable and shed light on the forces underlying the tradeoff between quality and total production, we restrict ourselves to a

[6] The assumption of spot transactions is central for this result: With commitment to long term contracts, the first best allocation can be achieved by backloading payments to a seller until the point when his reputation reaches a sufficiently high level. The combination of entry fees and sales subsidies in our model serves to replicate the same type of backloading. Regulation is therefore a remedy to the lack of commitment to long term contracts. MacLeod (2007) also presents results along these lines.

symmetric signal information structure, in which buyers learn through gradual accumulation of good and bad signals. In the laissez-faire equilibrium in this case, firms enter with low prices and revenues, then increasing or reducing their prices and revenues in a random fashion as good and bad signals arrive until enough bad signals accumulate to induce low quality firms to exit. A positive entry fee raises both quality and aggregate production at first, but once the fee is large enough, a tradeoff between quality and total production emerges, and further increases in entry fees eventually reduce welfare.

The entry fee has two opposing effects on the speed with which the market recognizes quality. On the one hand, the regulation reduces the proportion of entrants that are of low quality. But at the same time, a higher entry reputation also delays the exit by those low quality firms that do enter. The severity of the tradeoff between average quality and overall production depends on the strength of this second effect of entry fees on the exit rate and life expectancy of low quality entrants.

We illustrate this last point with two extreme alternative scenarios with asymmetric signals. In a *bad news* scenario, a low quality product is eventually revealed to be of low quality through the arrival of a perfectly revealing bad signal (such as cases of sudden safety hazards or product failures), forcing the firm to exit from the market. In this case, reputations improve as long as a firm has not experienced a bad signal, the exit rate and life expectancy of low quality firms is independent of their entry reputation, and entry fees unambiguously increase both quality and aggregate production such that low quality firms are completely driven out of the market. In a *good news* scenario, a high quality product is eventually revealed to be of high quality through the arrival of a perfectly revealing good signal (such as technological break-throughs). In this case, the higher entry reputations induced by an entry fee are almost completely offset by a corresponding decrease in exit rates of low quality firms, and the tradeoff between quality and aggregate production emerges even at the laissez-faire benchmark. Even in this case, however, small positive entry fees remain welfare improving.

Our analysis connects two mechanisms that provide firms with incentives to invest in quality: reputation and regulation. There is a rich literature studying each of these mechanisms in isolation, but to our knowledge they have so far not been systematically connected. In combining these mechanisms, we show how regulatory interventions can be used to leverage reputational incentives.

The literature on reputation concerns, surveyed recently in MacLeod (2007), views reputation as a valuable asset that the firm may lose if it is found to act opportunistically (Mailath and Samuelson (2001); Tadelis, 1999 and 2002). The firm's reputation is defined as outsider's beliefs about a hidden firm permanent characteristic, which is updated based on signals about the firm's performance.

We embed reputation dynamics in a general equilibrium model of firm dynamics, in which firms are free to enter or exit the market, and their initial investment determines the quality of their product. The proportion of firms that make the required upfront investments – and hence the reputation assigned to untested new entrants in equilibrium – adjusts endogenously to the firms' entry and investment incentives.[7] The resulting firm dynamics with free entry and exit then determine the resulting firm dynamics determine the number of high- and low-quality firms (and their respective reputations), and hence the severity of the adverse selection problem, in equilibrium. The general equilibrium model allows us to formalize the trade-off between average quality and total production that is at the heart of our analysis.

As a technical contribution, we offer a simple restriction on buyer beliefs under which the equilibrium is unique. In addition, we fully characterize equilibrium value functions, which makes the analysis of entry regulation especially tractable.[8]

The literature on regulation emphasizes how entry regulation can be welfare improving in settings with adverse selection and moral hazard. Klein and Leffler (1981, p. 168) highlight that the need to incentivize quality provision through quasi-rents ex post is inconsistent with free

[7]While the initial models considered exit to be exogenous, Hörner (2002), Bar-Isaac (2003), and Daley and Green (2012) introduce endogenous exit of firms, when these firms know their own type. None of these papers consider free entry or quality investments choices that endogenize initial reputations.

[8]In a previous NBER version of our paper (NBER Working Paper 17898) we characterize these value functions in continuous time as well. In contrast to the work of Prat and Alos-Ferrer (2012), Board and Meyer-ter-Vehn (2013) or Faingold and Sannikov (2011), who also exploit the tractability of continuous time, we derive value functions with both endogenous entry and exit.

entry of firms, unless these quasi-rents are dissipated at entry through advertisement, brand marketing or other forms of money-burning. However, since they present a partial equilibrium model without explicit consumer preferences or information, they do not explicitly discuss normative questions, welfare analysis, or regulatory interventions. Shapiro (1983) introduces a reputation formation interpretation of Klein and Leffler (1981, p. 168). He explicitly models consumers' preferences and shows that regulatory interventions that target quality directly, such as minimum quality standards, reduce the premium necessary to induce high quality. The welfare effects of such interventions trade off gains to consumers who highly value better quality provision against losses to consumers with low value for quality who are forced to pay for quality that they perceive as excessive.

We depart from this literature in two important respects. First, in our paper entry costs affect incentives for quality and thereby improve welfare, even if regulators are not able to identify or target quality.[9] This is consistent with the fact that most entry regulations in practice are not explicitly designed to screen entrants for quality.[10] Second, instead of focusing on a utilitarian welfare criterion with heterogeneous consumer preferences as in Shapiro (1983) and much of the related literature, our model admits a representative household interpretation. Regulation affects welfare through its impact on aggregate production and is therefore fully Pareto-improving.[11] Our analysis thus embeds the hold-up problem identified by Klein and Leffler (1981, p. 168) in a general equilibrium model of firm dynamics, which allows us to analyze

[9] Prescott and Townsend (1984) and Arnott, Greenwald, and Stiglitz (1994) discuss the scope for Pareto improving interventions, even if information imperfections cannot be directly overcome. In our model, the spot market outcome is not constrained Pareto optimal, because buyers and sellers are unable to commit to dynamic contracts with payments that differ from spot prices. Entry regulation is Pareto improving if it replicates the commitment to backloading of rewards that the market cannot generate on its own. This, of course, does not preclude the possibility of other market-based solutions to the commitment problem through, e.g., longer-term contracts, posting of bonds, or market-provided intermediation and certification services.

[10] Djankov et al. (2002) offer an extensive list of entry procedures which are required in many countries. The vast majority are related purely to screening and registration for tax purposes. To the extent that inspection and certification takes place, it is related to labor, work place safety, health and environmental standards, but not provider quality. Educational standards or professional licences that certify the operator's expertise are the main type of regulation that explicitly targets provider quality.

[11] In Leland (1979), Shaked and Sutton (1981) and Shapiro (1983 and 1986), entry costs improve welfare with asymmetric information if there is enough heterogeneity in how consumers value quality and regulation improves provision of quality to those consumers that value it highly. But these are not Pareto improvements since those consumers who prefer low quality are harmed. Similarly, in Garcia-Fontes and Hopenhayn (2000) entry restrictions can improve welfare by their effect on the market size, but because the pool of consumers, heterogeneous in their preferences, change.

regulatory interventions from a macro-economic perspective.

In the following section, we describe the economy and characterize the spot market equilibrium for two extreme benchmarks: full information and no learning. In Section III we characterize the spot market equilibrium in steady-state with imperfectly informative signals. In Section IV we discuss the effects of regulation on equilibrium outcomes. Section V concludes. The Appendix contains the proofs.

II. The Model

In this section, we describe the economic environment, define a spot market equilibrium with regulation, and show that under full information, the spot market equilibrium without regulation implements the socially optimal allocation.

II.A. The Economy

Time is discrete with time periods numbered $t = 0, 1, 2, \ldots$. At each time t, consumers derive utility from a *final good*, whose consumption is denoted by Y_t, and a *numeraire good*, whose consumption is denoted by N_t. These goods are non-storable. Consumer preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left(U(Y_t) + N_t \right), \tag{1}$$

where $U' > 0$, $U'' < 0$, $U'(0) = \infty$, $\lim_{Y \to \infty} U'(Y) = 0$, and $\beta \in (0, 1)$.

At each time t, there is a non-storable endowment of 1 unit of the numeraire good. The final good is produced with a constant returns to scale technology that uses produced intermediate goods as the only inputs. These intermediate goods are of uncertain quality and we refer to them throughout as *experience goods* because there will be learning through experiencing their quality.[12]

[12]Our interpretation of trade being carried out between producers of differentiated intermediate goods and producers who use those intermediate goods to produce final consumption is standard in the international trade literature. See, for example, Ethier (1982). The key assumption regarding trade in our model is that individual buyers and sellers do not engage in long-term trading relationships.

Each period, there is a continuum of *experience good firms* in the economy, each with a capacity to produce 1 unit of the experience good per period at zero marginal cost for as long as that firm remains active. Experience good firms can become inactive and *exit* either by the owner's decision, or for exogenous, stochastic reasons.[13] Experience good firms that exit at t cannot return to production at later dates.

New experience good firms *enter* as high-quality (H) or low-quality (L) firms. An entering firm must invest K units of the numeraire good to be of high quality. Low-quality firms can enter at zero cost. We denote the measure of new experience good firms entering at t by $m_t^e \geq 0$. The fraction of those entrants who invest to become high-quality is denoted $\phi_t^e \in [0,1]$. The total numeraire good invested in the creation of new high-quality firms in period t is therefore $K\phi_t^e m_t^e$, and the resource constraint for the numeraire good is

$$N_t + K\phi_t^e m_t^e = 1. \tag{2}$$

The quality of an experience good firm determines the expected productivity of the experience good produced by that firm in use as an input to produce the final good. One unit of output from a high-quality experience good firm contributes in expectation $y(1) > 0$ units of final goods at the margin, whereas one unit of output from a low-quality experience good firm contributes in expectation $y(0) < 0$ units of final goods at the margin.[14] If m_{Ht} and m_{Lt} denote the measures of high- and low-quality firms active in period t, then aggregate production of the final good in period t is

$$Y_t = y(1)m_{Ht} + y(0)m_{Lt}. \tag{3}$$

By assuming that experience goods are perfectly substitutable in the production of the final consumption good, we obtain the result that the buyers of these intermediates are competitive

[13]The assumption of stochastic exit ensures that there is ongoing entry of new firms in steady-state.

[14]The assumptions of zero marginal cost of production for the intermediate good and a negative marginal product of low-quality intermediate goods are normalizations that simplify the exposition, but do not affect the main results.

final goods firms that have a common valuation and are risk neutral (have constant marginal valuations of an additional unit of the experience good). At the same time, we have a final consumer in our model economy with diminishing marginal utility for aggregate output of the final consumption good generated from these experience goods. These assumptions allow us to measure social welfare based on the consumer surplus of a representative household. In particular, we abstract from the matching and aggregation issues that would arise if individual producers with different expected qualities sold to individual consumers with diverse valuations for the qualities of the experience good.

Each experience good firm is characterized by its *reputation* $\phi \in [0,1]$, which is defined as the buyers' public belief (probability) that the firm is of high quality. The expected contribution to production of the final good by the output of an experience good firm with reputation ϕ is denoted $y(\phi)$ and given by

$$y(\phi) = \phi y(1) + (1-\phi)y(0). \tag{4}$$

Each firm's reputation evolves over time through the observation of exogenous signals about its quality and the firm owner's decision to remain active, which may be viewed as an endogenous signal of its quality.

The timing of events within a period is as follows. At the beginning of a period t, new experience good firms enter and choose whether to invest in quality. They all start with a reputation ϕ_t^e that is equal to the fraction of entering firms that invest in high quality. All firms (including entrants) then decide, based on their reputation $\phi \in [0,1]$, whether to exit or continue. We denote by $\omega_{it}(\phi) \in [0,\bar{\omega}]$ the probability that a firm of quality $i = \{L,H\}$ and reputation ϕ stays in the market; with probability $1-\omega_{it}(\phi)$, this firm exits. The upper bound $\bar{\omega} \in (0,1)$ denotes the maximal survival rate, and then $1-\bar{\omega}$ the exogenous rate of exit.[15] Buyers (final good producers) form *interim beliefs* $\phi_t^c(\phi) \in [0,1]$ about the quality of those firms that, for a given ϕ, chose

[15] The assumption that new entrants may choose to immediately exit (or be forced to do so due to a random exit shock) is only done for analytical convenience, as this allows us to treat the decision problems for entrants and incumbents in a completely symmetric fashion.

to continue. These interim beliefs satisfy Bayes' Rule whenever applicable. Given continuation strategies $\{\omega_{Lt}(\cdot), \omega_{Ht}(\cdot)\}$, $\phi_t^c(\cdot)$ takes the form

$$\phi_t^c(\phi) = \frac{\phi\omega_{Ht}(\phi)}{\phi\omega_{Ht}(\phi) + (1-\phi)\omega_{Lt}(\phi)}, \tag{5}$$

whenever $\max\{\omega_{Ht}(\phi), \omega_{Lt}(\phi)\} > 0$.

Production and trade then occurs at these interim beliefs, with the owners of active firms selling their output to final good producers. After production and trade have taken place, public signals about the quality of each experience good firm are realized, and the firms' reputations are updated to start the subsequent period $t+1$.

We consider two versions of the signal structure that governs the evolution of firms' reputations over time. In a *Bernoulli trials* version, a *good* or *bad* public signal is revealed each period after trade for each active experience good firm. These signals lead to updating of reputations to ϕ_{t+1} to start period $t+1$ for all firms that were active in period t. We denote by α_i the probability that an active experience good firm of type $i \in \{H, L\}$ generates a good signal, and assume without loss of generality that $\alpha_H > \alpha_L$. Given interim beliefs $\phi_t^c(\phi)$, the firm's reputation entering period $t+1$ is governed by Bayes' Rule and either

$$\phi_{t+1} = \phi^g(\phi_t^c(\phi)) = \frac{\phi_t^c(\phi)\alpha_H}{\phi_t^c(\phi)\alpha_H + (1-\phi_t^c(\phi))\alpha_L} \tag{6}$$

or

$$\phi_{t+1} = \phi^b(\phi_t^c(\phi)) = \frac{\phi_t^c(\phi)(1-\alpha_H)}{\phi_t^c(\phi)(1-\alpha_H) + (1-\phi_t^c(\phi))(1-\alpha_L)}, \tag{7}$$

depending on whether a good or a bad signal is realized, respectively.

In a *Single trial* version, such a signal is realized only for firms that newly entered. No further signals are revealed about incumbent firms. A firm that enters in period t with reputation ϕ_t^e will start period $t+1$ with a reputation of $\phi^g(\phi_t^c(\phi_t^e))$ or $\phi^b(\phi_t^c(\phi_t^e))$ depending on the realized signal, and will retain this reputation for as long as it remains in production. This signal structure is similar to that in Klein and Leffler (1981) and makes the equilibrium characterization especially tractable.

The Bernoulli trials version is substantially more involved, so we focus on a *symmetric signals* case, $\alpha_H = 1 - \alpha_L$, which implies that $\phi^g(\phi^b(\phi)) = \phi$, i.e. a good signal and a bad signal exactly offset each other. Details of the analysis for this case are given in Appendix B.

Denote by $\nu_{it}(\phi)$, for $i \in \{H, L\}$, the measures of high- and low quality firms after entry has occurred but before exit. The measures of high- and low-quality firms active in production satisfy $m_{it} = \int_\phi \omega_{it}(\phi) d\nu_{it}(\phi)$. The evolution of these measures from one period to the next is determined in the standard way. First, firms continuation strategies $\omega_{it}(\phi)$ reduce the measure of firms through exit. The reputations of those firms that continue are updated according to buyers' interim beliefs $\phi_t^c(\phi)$. Second, trade occurs, signals are observed, and active firms' reputations are updated according to Bayes' rule in (6) and (7). Third, a measure m_{t+1}^e of firms enters at the beginning of $t+1$ with reputation ϕ_{t+1}^e, with a fraction ϕ_{t+1}^e of those firms being high-quality and $1 - \phi_{t+1}^e$ low-quality. These steps define ν_{Ht+1} and ν_{Lt+1}.

An *allocation* in this environment is a sequence of consumption of the final and numeraire goods $\{Y_t, N_t\}$, measures and initial reputations for entrants $\{m_t^e, \phi_t^e\}$, buyers' interim beliefs $\{\phi_t^c(\phi)\}$, and, continuation strategies $\{\omega_{it}(\phi)\}$, reputational distributions $\{\nu_{it}(\phi)\}$, and measures of active firms $\{m_{it}\}$ for $i = \{L, H\}$.

An allocation is *feasible* if it satisfies the numeraire good and the final good resource constraints (2) and (3), the constraints on the evolution of the measure of firms by reputation implied by continuation strategies, buyers' interim beliefs, and measures of entry and initial reputations for entrants. An allocation is *stationary* if it constant in t. When referring to stationary allocations, we suppress the time subscript.

II.B. Stationary Spot Market Equilibrium

We now consider stationary equilibrium allocations in an economy in which all transactions take place at spot prices. The final good producers buy experience goods at a *spot market price*

$p(\phi)$ (in terms of the numeraire good) from an experience good firm with reputation ϕ, and consumers buy the final good at a spot market price P. Within this market we also allow for the use of two simple regulatory tools: (i) a tax $F \geq 0$ that is imposed on new experience good firms entering the market, and (ii) a subsidy s per unit of the final good purchased. The net proceeds of these taxes and subsidies are rebated to consumers or financed through lump sum taxes on consumers. We focus on a *stationary spot market equilibrium*, in which all prices, quantities and regulatory tools are constant over time.

The stationary final good price P is given by the marginal utility of the final good, adjusted for the sales subsidy:

(8)
$$P = (1+s)U'(Y).$$

The spot market price $p(\phi)$ for an experience good firm with reputation ϕ is given by

(9)
$$p(\phi) = y(\phi)P,$$

and is thus the product of the final good price P and the expected marginal product $y(\phi)$ of the experience good in the production of the final good with expectations based on the reputation of the selling firm.[16] $p(\phi)/P = y(\phi)$ then corresponds to the flow of *normalized* profits from an active experience good firm with reputation ϕ.

We let $V_i(\phi^e)$ denote the discounted expected value of *normalized profits* for an experience good firm of quality $i \in \{H, L\}$ that enters with reputation ϕ^e. In the Bernoulli trials version of the model, the value for a new entrant is the same as the beginning-of-period value of an incumbent with the same reputation, so we can define $V_i(\phi)$ recursively. Taking buyers' interim beliefs

[16] Competition among final good producers in buying the experience goods drives the spot market prices to their reservation value. Hence experience good producers obtain quasi-rents once they gain a sufficiently high reputation. In equilibrium, these quasi-rents are dissipated, exactly compensating firms for their initial investment K (for high quality firms), regulatory entry costs F, and initial period of sales at prices below marginal cost when they have initially low reputation.

based on continuation $\phi^c(\phi)$ as given, $V_i(\phi)$ is the unique solution to the Bellman equation

$$V_i(\phi) = \max_{\omega \in [0,\bar{\omega}]} \omega \left[y(\phi^c(\phi)) + \beta \left(\alpha_i V_i(\phi^g(\phi^c(\phi))) + (1-\alpha_i) V_i(\phi^b(\phi^c(\phi))) \right) \right], \tag{10}$$

where $\phi^g(\cdot)$ and $\phi^b(\cdot)$ are defined by (6) and (7). This Bellman equation (10) also defines the set of optimal continuation strategies for a firm of quality i given buyers' interim beliefs. A continuation strategy $\omega_i(\phi)$ is a *best response* to buyers' interim beliefs $\phi^c(\phi)$ only if $\omega_i(\phi) = \bar{\omega}$ when $V_i(\phi) > 0$.

In the single trial version of the model, we work backwards to define the normalized value functions for entrants, $V_i(\phi)$. Let

$$\widehat{V}(\phi) = \max_{\omega \in [0,\bar{\omega}]} \omega \left[y(\phi^c(\phi)) + \beta \widehat{V}(\phi^c(\phi)) \right]$$

denote the normalized value of an incumbent firm that has already experienced its signal and is now choosing whether to stay in the market. This is independent of the firm's quality since no further signals will separate a high quality firm from its low quality peers. The firms' normalized value at entry, $V_i(\phi)$, is then given by

$$V_i(\phi) = \max_{\omega \in [0,\bar{\omega}]} \omega \left[y(\phi^c(\phi)) + \beta \left(\alpha_i \widehat{V}(\phi^g(\phi^c(\phi))) + (1-\alpha_i) \widehat{V}(\phi^b(\phi^c(\phi))) \right) \right]. \tag{11}$$

Expressed in terms of the numeraire good, an experience good firms' value at entry with quality $i \in \{H, L\}$ and reputation ϕ^e is $P \cdot V_i(\phi^e)$. In equilibrium, profits at entry must be non-positive for both high- and low-quality firms . With a stationary subsidy s to sales of the final good and entry fees of size F, this requirement is

$$(1+s)U'(Y)V_H(\phi^e) - K - F \leq 0, \tag{12}$$

with equality if $\phi^e m^e > 0$, and

$$(1+s)U'(Y)V_L(\phi^e) - F \leq 0, \tag{13}$$

with equality if $(1-\phi^e)m^e > 0$.

We conclude this sub-section with the definition of a *stationary spot market equilibrium.*

Definition 1 *Stationary spot market equilibrium*

A stationary spot market equilibrium consists of a feasible, stationary allocation $\{Y, N, m^e, \phi^e, \phi^c(\phi), \omega_i(\phi), \nu_i(\phi), m_i\}$, *buyers' interim beliefs* $\phi^c(\cdot)$, *and normalized value functions* $\{V_i(\phi)\}$ *defined as in (10) or (11) such that*

(i) *The continuation strategies* $\omega_i(\phi)$ *are a best response to buyers' interim beliefs* ϕ^c.

(ii) *Buyers' interim beliefs* $\phi^c(\phi)$ *are consistent with the continuation strategies* $\omega_i(\phi)$ *as in (5) where Bayes' rule is defined, and*

(iii) *The zero profits on entry conditions (12) and (13) are satisfied.*

In what follows, we refer to the equilibrium with $F = 0$ and $s = 0$ as the *laissez-faire* benchmark without any regulatory interventions. We denote by ϕ^e_{lf} and Y_{lf} the entry reputation and final good production in the laissez-faire benchmark.

II.C. Benchmarks: Social Optimum and Spot Market Equilibrium

Full information: For further reference, we briefly discuss the optimality conditions and spot market equilibrium governing firm entry and output of the final good in a stationary equilibrium with full information. In our model, high quality experience good firms are directly analogous to units of capital in the neo-classical growth model. Therefore, following standard arguments the laissez-faire equilibrium attains a socially optimal level of entry and production if quality investments are perfectly observable at the time of entry. No regulation in the form of entry fees for experience good firms or subsidies to final good sales can improve welfare.

Equating the marginal social cost of creating a new high quality firm, K, to the marginal benefit in a stationary allocation gives

$$\frac{\bar{\omega}}{1-\beta\bar{\omega}} y(1) U'(\bar{Y}) = K. \tag{14}$$

where $\bar{Y}$ denotes the stationary production of the final good with full information. The optimal stock of high-quality experience good firms is the stock required to produce stationary output of the final good $\bar{Y}$ as determined by equation (14). Likewise, the optimal rate of entry of high quality firms is the rate necessary to maintain that stock and the optimal fraction of high quality entrants is $\phi^e = 1$.

These allocations are implemented by a stationary spot market equilibrium with spot prices $\bar{p}(\phi) = y(\phi)U'(\bar{Y})$ and $\bar{P} = U'(\bar{Y})$ for intermediate goods produced by firms with reputation ϕ and for the final good, respectively. Since $\bar{p}\,(0) < 0$, low quality firms never have an incentive to enter the market. The normalized value associated with a high-quality firm at these spot prices is

$$V_H(1) = \frac{\bar{\omega}}{1-\beta\bar{\omega}} y(1) > 0,$$

and equilibrium entry by high quality firms is characterized by the free entry condition $U'(\bar{Y})V_H(1) = K$, coinciding with the socially optimal entry and production.[17]

No information: At the other extreme, if investment is non-observable and signals remain uninformative ($\alpha_H = \alpha_L$), the adverse selection problem associated with free entry of low-quality firms is so severe that there is no production of the final good in steady state. If signals are uninformative, then they do not generate updating of buyers' beliefs, i.e. $\phi^g(\phi^c) = \phi^b(\phi^c) = \phi^c$, and it is straight-forward to check that for any interim belief function $\phi^c(\cdot)$, the normalized value functions for the two types must be the same, i.e. $V_H(\phi) = V_L(\phi)$. But then given the entry condition for low quality firms (13), it is impossible to offer high quality experience good firms a sufficient reward for their investment to enter the market, and therefore $\phi^e m^e = 0$, i.e. no

[17] The same argument extends to non-stationary allocations starting from an arbitrary initial distribution of experience good firms.

high quality firm entering the market. But once high quality firms have no incentive to enter, the stationary equilibrium allocation must have no entry of firms at all and hence there can be no positive production of the final good once the initial stock of high-quality firms dies out. This argument applies irrespective of the choice of regulatory policies F and s, implying that regulatory interventions have no effect, if buyers do not have any access to information about seller quality.

III. Equilibrium with Imperfectly Informative Signals

We now construct a steady-state spot market equilibrium in our model for both the Single Trial and Bernoulli Trial versions of the model. We show that there is a generic multiplicity of equilibria in our model associated with different configurations of buyers' interim beliefs about firms' continuation decisions as summarized in $\phi^c(\phi)$. We then present restrictions on those beliefs that ensure in the Bernoulli trials version of the model that this equilibrium is unique.

III.A. Equilibrium Characterization

In the equilibrium that we construct, buyers beliefs are characterized by a threshold reputation $\underline{\phi}$ such that $\phi^c(\phi) = \phi$ for $\phi > \underline{\phi}$ and $\phi^c(\phi) = \underline{\phi}$ for $0 < \phi \leq \underline{\phi}$. In equilibrium, we refer to this threshold reputation $\underline{\phi}$ as the *exit threshold for low quality firms* as it is the highest reputation for which low quality firms will be willing to exit endogenously. These beliefs are confirmed by continuation strategies for high and low quality firms in which high quality firms choose to continue to the extent possible for all $\phi > 0$, ($\omega_H(\phi) = \bar{\omega}$), and low quality firms do the same for $\phi \geq \underline{\phi}$ and otherwise randomize over continuation and exit with $\omega_L(\phi) < \bar{\omega}$ set so that Bayes Rule in (5) implies $\phi^c(\phi) = \underline{\phi}$ for $\phi < \underline{\phi}$. Therefore, in our equilibrium, all active firms' interim reputations remain in an interval $[\underline{\phi}, 1]$.

In the *Single Trial* version of the model, with interim beliefs $\phi^c(\phi) = \max\left\{\phi, \underline{\phi}\right\}$, we have $\widehat{V}(\phi) = \bar{\omega}/\left(1 - \beta\bar{\omega}\right) \cdot \max\left\{0, y\left(\max\left\{\phi, \underline{\phi}\right\}\right)\right\}$ and the exit threshold for low quality firms $\underline{\phi}$ must satisfy $y(\underline{\phi}) = 0$. Both high and low quality firms earn zero profits when $\phi \leq \underline{\phi}$, even if they randomize their continuation decision and remain active, as conjectured above. The normalized value at entry then satisfies (11)for $i \in \{H, L\}$, i.e. high and low-quality entrants, with $\widehat{V}(\phi)$ as above and $\phi^c(\phi) = \max\left\{\phi, \underline{\phi}\right\}$.

To construct the exit threshold $\underline{\phi}$ in the *Bernoulli Trials* version of the model, consider first the solution $V_L^*(\cdot)$ to the following auxiliary Bellman equation:

$$V_L^*(\phi) = \max_{\omega \in [0,\bar{\omega})]} \omega\left[y(\phi) + \beta\left(\alpha_L V_L^*(\phi^g(\phi)) + (1-\alpha_L)V_L^*(\phi^b(\phi))\right)\right]. \tag{15}$$

This auxiliary Bellman equation corresponds to a standard exit option problem of a low-quality firm that faces beliefs $\phi^c(\phi) = \phi$ for all ϕ. The solution to this problem defines a unique optimal exit threshold $\underline{\phi} \in (0,1)$ such that $V_L^*(\phi)$ is positive and strictly increasing for all $\phi > \underline{\phi}$ and $V_L^*(\phi) = 0$ for $\phi \leq \underline{\phi}$. With this construction of $\underline{\phi}$, it's a straight-forward matter to check that the function $V_L^*(\cdot)$ also solves the original Bellman equation (10) with buyers' continuation beliefs equal to $\phi^c(\phi) = \max\left\{\phi, \underline{\phi}\right\}$. That is, when facing beliefs $\phi^c(\phi) = \max\{\phi, \underline{\phi}\}$, a low quality firm would optimally chose to remain active as long as $\phi > \underline{\phi}$, but exit with positive probability once its reputation level drops below the threshold $\underline{\phi}$. Moreover, given these buyers' beliefs we compute $V_H(\cdot)$, and since $V_H(\phi) > V_L(\phi) \geq 0$, high quality firms always strive to continue, as conjectured above.

Figures I and II illustrate these first two steps of constructing buyers' beliefs and firms' value functions and continuation decisions for the Bernoulli trials case.[18]

In both versions of the model, the free entry conditions (12) and (13) for high and low quality firms must hold with equality. It follows from our regularity conditions on $U(\cdot)$ that these free entry conditions determine a unique well defined equilibrium entry reputation ϕ^e and production of the final good Y. Given the hypothesized entry and continuation strategies for high and low quality firms, it is then a straightforward calculation to compute the entry rate m^e and the consumption of the numeraire good N consistent with ϕ^e and Y.

To summarize, we have established the following result:

[18]The figures were constructed assuming the continuous time limit and a signal structure that follows a Brownian motion process.

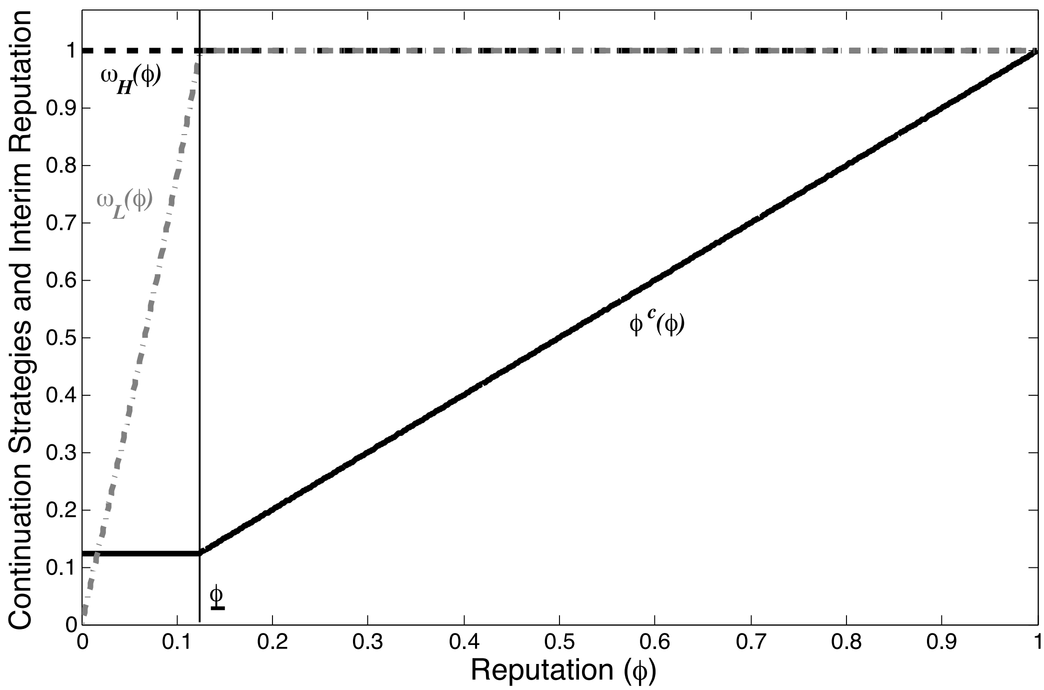

Figure I: Continuation Decisions and Interim Reputation

Proposition 1 *Stationary spot market equilibrium with Bernoulli Trials.*

Let $\underline{\phi}$ be defined by $V_L(\underline{\phi}) = 0$ where $V_L(\cdot)$ solves Bellman equation (15), and let $V_H(\cdot)$ be defined as the solution to (10), with $\phi^c(\phi) = \max\left\{\phi, \underline{\phi}\right\}$. Given regulatory policies $F \geq 0$ and $s \geq 0$, there exists a stationary spot market equilibrium in which:

(i) *High quality firms always strive to continue ($\omega_H(\phi) = \bar{\omega}$). Low quality firms strive to continue ($\omega_L(\phi) = \bar{\omega}$) if $\phi \geq \underline{\phi}$, but randomize over continuation ($\omega_L(\phi) < \bar{\omega}$) when $\phi < \underline{\phi}$.*

(ii) *Buyers' interim beliefs are given by $\phi^c(\phi) = \max\left\{\phi, \underline{\phi}\right\}$.*

(iii) *The entry reputation and steady-state output are determined by free entry (12) and (13).*

The same proposition applies to the single trial version after adjusting for the construction of the equilibrium value functions.

Equilibrium Uniqueness. The equilibrium we have constructed above is not unique without further restrictions on buyers' beliefs. In particular, in the Single Trial version of the model, there is an alternative configuration of beliefs with $\phi^c(\phi) = \phi$ for all ϕ and continuation strate-

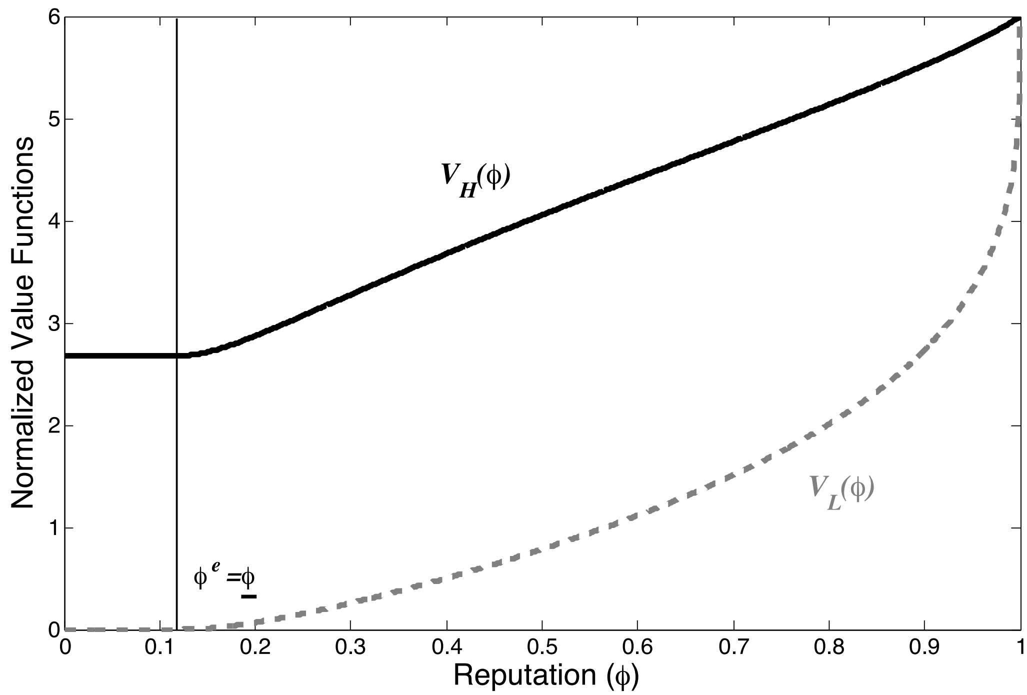

Figure II: Value Functions: Brownian diffusion

gies $\omega_i(\phi) = \bar{\omega}$ for $\phi \geq \underline{\phi}$ and $\omega_i(\phi) = 0$, which is otherwise consistent with the same solutions for $V_i(\phi)$, $\underline{\phi}$, ϕ^e, and Y, but different solutions for N and m^e since these quantities are impacted by the exit decisions of high quality firms that receive the bad signal, $\omega_H(\phi^b(\phi^e))$. An equilibrium with $\phi^c(\phi) = \phi$ for all ϕ can also be sustained in the Bernoulli Trials version of the model, if $\phi^b(\underline{\phi})$ is low enough so that high quality firms with reputation $\phi^b(\underline{\phi})$ have negative discounted present values of profits when facing interim beliefs $\phi^c(\phi) = \phi$ for all ϕ. These equilibria feature socially inefficient exit by high quality firms.

Our next result shows that, if time periods in the model are "sufficiently short", the equilibrium characterized by proposition 1 is the unique equilibrium in which buyers' interim beliefs satisfy the (arguably reasonable) restriction that continuation is seen as a positive signal of firm quality. To state our result, let $\underline{\phi}$ denote the low quality firms' exit threshold in the equilibrium characterized by proposition 1, with $\phi^c(\phi) = \max\left\{\underline{\phi}, \phi\right\}$. Let $V_H^*(\cdot)$ the value function of a high quality firm, when facing interim beliefs $\phi^c(\phi) = \phi$ for all ϕ.

Proposition 2 ***Equilibrium Uniqueness:*** *Suppose that* $y\left(0\right)+\beta\alpha_H V_H^*\left(\underline{\phi}\right) \geq 0$. *Then, in the Bernoulli* *trials version of the model, the equilibrium characterized in proposition 1 is the unique stationary spot market equilibrium with positive entry in which "Continuation signals Quality": $\phi^c(\phi) \geq \phi$ for all $\phi \in [0,1]$.*

The condition $y(0) + \beta\alpha_H V_H^*(\underline{\phi}) \geq 0$ compares the one period flow payoff of a firm with the worst reputation, $y(0)$, against the expected NPV of discounted future profits $\beta V_H^*(\underline{\phi})$ of a high-quality firm with a reputation at $\underline{\phi}$, scaled by the high quality firm's probability of a good signal α_H. Therefore, unless the arrival rate of a good signal α_H also scales with calendar time, this condition is automatically satisfied when time periods are sufficiently short in calendar time.

The restriction on beliefs that "Continuation signals Quality" can be motivated in several ways.[19] Suppose for example that any firm that wishes to exit is unable to do so with some small type-independent probability. This perturbation determines continuation beliefs by $\phi^c(\phi) = \phi$ at any ϕ such that both firms wish to exit for sure, and thus corresponds to a special case of our criterion.[20] But the criterion also resembles forward induction criteria, according to which continuation beliefs should favor those types that have more to gain from continuation, which in our case would be the high type firms.

Our uniqueness result is based on an infection logic for which "Continuation signals Quality" is crucial. We first show that if $\phi^c(\phi) \geq \phi$, any firm with a reputation $\phi \geq \underline{\phi}$ does not have an incentive to exit immediately, regardless of the equilibrium played, or their type. This in turn determines $\phi^c(\phi) = \phi$ for $\phi \geq \underline{\phi}$. Then, if ϕ is sufficiently close to $\underline{\phi}$ and $\phi^c(\phi) \geq \phi$, a single good signal will suffice to push the reputation above $\underline{\phi}$ ($\phi^g(\phi) \geq \underline{\phi}$), giving high types a strict incentive to continue at any equilibrium. But then low type firms will randomize over their exit, which pins the continuation beliefs at $\underline{\phi}$ for any ϕ s.t. $\phi^g(\phi) \geq \underline{\phi}$. But then, high types with slightly lower reputations also recognize that a single positive signal will suffice to push the continuation reputation up to $\underline{\phi}$, which in turn gives these firms a strict incentive to continue etc. If $y(0) + \beta\alpha_H V_H^*(\underline{\phi}) \geq 0$, the same argument eventually applies to any strictly positive

[19] An equivalent condition is also imposed in related work by Daley and Green (2012, section 5) who refer to it as "belief monotonicity".
[20] See Bar-Isaac (2003, section 6) for an analysis of this perturbation in a similar model.

reputation level, so that $\phi^c(\phi)$ is uniquely pinned down for $\phi > 0$.[21]

In addition to equilibria with inefficient exit, there also exist equilibria with no entry, which are sustained by an entry reputation of $\phi^c(\phi^e) = \phi^e = 0$ that is never "tested" in equilibrium. If $y(0) + \beta\alpha_H V_H^*(\underline{\phi}) \geq 0$, these equilibria are not robust to entry by a small measure of high quality firms, since this would result in a strictly positive entry reputation $\phi^e > 0$ and $\phi^c(\phi^e) = \underline{\phi}$, by the preceding argument.

This proposition applies only to the Bernoulli Trials version of the model, not the single signal version. To obtain that high quality firms have a strict incentive to continue for reputation levels close to $\underline{\phi}$, we have made use of the fact that $V_H(\underline{\phi}) > V_L(\underline{\phi}) = 0$ whenever both firms continue for any $\phi \geq \underline{\phi}$. This eventually determines continuation incentives for high quality firms at all reputation levels. In the single signal version instead, the value functions are the same, so there is always an equilibrium in which both firms exit at the same threshold reputation.[22]

III.B. The Laissez-faire Benchmark

The entry reputation ϕ^e_{lf} in the laissez-faire benchmark is derived from the free entry condition for low quality firms: $V_L(\phi^e_{lf}) = 0$. In the Bernoulli Trials version, this implies that the entry and exit reputation thresholds are the same, i.e. $\phi^e_{lf} = \underline{\phi}$. In the Single Trials version, it must be the case that $\underline{\phi} > \phi^e_{lf}$ so that low quality firms do not earn positive profits upon entry.

Given ϕ^e_{lf}, the level of final good production Y_{lf} at the laissez-faire equilibrium is derived from the free entry condition for high quality firms: $V_H(\phi^e_{lf})U'(Y_{lf}) = K$. Because $V_H(\phi^e_{lf}) < V_H(1)$, we have the following result for both versions of our model:

Proposition 3 *The steady-state level of the final good output in the laissez-faire equilibrium is lower than that in the full information benchmark. That is,* $Y_{lf} < \bar{Y}$.

[21] The condition $y(0) + \beta\alpha_H V_H^*(\underline{\phi}) \geq 0$ can be further relaxed at the cost of additional notation by recognizing that along the iteration the the "worst" sustainable interim belief $\phi^c(\cdot)$ is gradually improved relative to $\phi^c(\phi) = \phi$. However, the general argument of comparing a "worst" one-period payoff against the expected NPV after a single positive signal remains exactly the same.

[22] The logic of this argument suggests that a uniqueness result can be established as long as additional information arrives with positive probability to ensure that $V_H(\underline{\phi}) > V_L(\underline{\phi}) = 0$, or that high quality firms have some other intrinsic reason to stay in the market.

This proposition shows that, although reputation mitigates the lemons problem and allows for some positive production of the final good (relative to the no information benchmark), production is limited by the fact that high-quality firms can no longer appropriate the full marginal value of their investments, as they need to endure lower profits after entry while they accumulate a good reputation. This reduces welfare relative to the full information benchmark.

Consider now how the informativeness of the signal impacts the equilibrium. The ratio α_H/α_L determines the informativeness of a good signal about quality; notice that $\phi^g(\phi^e)$ is increasing in α_H/α_L for a fixed value of ϕ^e. The ratio $(1-\alpha_L)/(1-\alpha_H)$ determines the informativeness of a bad signal about quality; $\phi^b(\phi^e)$ is decreasing in $(1-\alpha_L)/(1-\alpha_H)$ for a fixed value of ϕ^e. Both signals thus become more informative as α_H increases or α_L decreases. In the single trial version, we therefore say that signals become more informative if α_H increases or α_L declines, holding the other signal probability fixed. In the Bernoulli version, signals become more informative if α_H increases and (at the same time) $\alpha_L = 1-\alpha_H$ declines. For both versions of our model we show the following proposition:

Proposition 4 *The laissez-faire output Y_{lf} increases monotonically with the informativeness of signals.*

The key to the result is to show that at the laissez-faire benchmark the quasi-rents of high-quality firms at entry, $V_H(\phi^e)$, are an increasing function of signal informativeness. While it is quite intuitive that $V_H(\phi^e)$ increases with signal informativeness for a given ϕ^e, the equilibrium entry reputation also changes with the signal structure and turns out to do so non-monotonically. The low quality firms' incentives to enter depend on the upside chance of enjoying false positive signals. But this upside chance exists neither for extremely noisy information structures (in which case the market will simply not update, so the low quality entrant has nothing to gain from a good signal), nor for extremely informative signal structures (in which case a low quality entrant would have a lot to gain from favorable signals but the chance of such a signal occurring vanishes). The low quality firms' incentive to enter is therefore strongest with intermediate signal structures, and the entry reputation is typically a non-monotonic function of signal informativeness, and lowest at intermediate levels.

Because of this non-monotonicity, we have not been able to offer a general statement on which of these forces dominate without more specific assumptions about the signal structure. In the specific signal structure we assume, however, we show that the direct effect of better signal quality always outweighs the indirect effect of low entry reputations, and hence laissez-faire output Y_{lf} is always increasing in signal quality.

IV. The Impact of Regulation on Equilibrium

What is the impact of regulatory entry costs F and sales subsidies s on the stationary equilibrium quality of entrants ϕ^e and the volume of production of the final good Y? We answer this question by analyzing the impact of these regulatory instruments on the free entry conditions for high and low quality firms. Given specific policies $s > 0$ and $F > 0$, the free entry conditions for high and low quality firms define two schedules of Y as a function of ϕ^e that we must solve jointly to find the equilibrium values of ϕ^e and Y. Thus, the existence of an equilibrium and comparative statics of equilibrium ϕ^e and Y with respect to policies depend on the specific shapes of these two schedules.

Consider first how the choice of a regulatory entry cost $F > 0$ impacts the equilibrium entry reputation ϕ^e. Taking the ratio of the free entry conditions, ϕ^e must satisfy

$$\frac{V_L(\phi^e)}{V_H(\phi^e)} = \frac{F}{F+K}, \tag{16}$$

which is independent of Y. Therefore, the impact of F on the equilibrium entry reputation ϕ^e is determined by the ratio of low to high quality firm's value functions. The right hand side of equation (16) is a continuous, strictly increasing function of F, rising from 0 at $F = 0$ to 1 as F approaches infinity. The left hand side of this expression is equal to zero at the laissez-faire benchmark ($\phi^e = \phi^e_{lf}$), always strictly less than 1, and converges to one when ϕ^e converges to 1 and $\alpha_H < 1$. In this case, for any $\phi^e \in (\phi^e_{lf}, 1)$ desired by the regulator, there exists an entry fee $F > 0$ for which the equilibrium entry reputation is ϕ^e desired by the regulator, and for each F there exists (at least) one corresponding entry reputation ϕ^e, whenever $\alpha_H < 1$. The entry reputation is unique and strictly monotonic in F whenever $V_L(\phi^e)/V_H(\phi^e)$ is strictly monotonic in ϕ^e. In the Single Trial version of the model, since $y(\phi)$ is linear in ϕ, this is always the case, but we have not been able to show a corresponding result for the Bernoulli Trials version, except for the continuous time limits.[23]

Equation (16) illustrates how the entry fee affects incentives for quality investments. For a given entry reputation ϕ^e, the value $V_L(\phi^e)$ corresponds to the surplus or information rent that a firm can extract by entering the market as an undetected low type. At the laissez-faire equilibrium, free entry leads to full dissipation of this information rent. With an entry fee, low quality firms must earn higher information rents, and these rents are no longer dissipated, but instead taxed away through the entry fee. At equilibrium, the increase in information rents results from a shift in the composition of new entrants: a higher quasi-rent for low quality firms upon entry is possible only if they enter in smaller proportion and with a higher entry reputation – and hence a higher average quality of entrants.[24]

Now consider how the sales subsidy s impacts the steady-state production of the final good Y given an entry reputation ϕ^e corresponding to a given choice of a regulatory entry fee $F \geq 0$. Taking the difference of the free entry conditions for high and low quality firms, we have that ϕ^e and Y must satisfy

(17)
$$V_H(\phi^e) - V_L(\phi^e) = \frac{K}{(1+s)U'(Y)}.$$

This condition states that the price of final output (including the sales subsidy) must adjust to compensate high quality entrants for the extra cost they incur upon entering the market with high quality. The entry fee F thus affects final goods production Y and the final good price $P = U'(Y)$ only indirectly through the effect of the corresponding entry reputation level ϕ^e on the difference between the value functions $V_H(\phi^e) - V_L(\phi^e)$. For a given P, this gap governs

[23] See our earlier version of this paper (NBER Working Paper 17898) for a treatment of our model in continuous time.

[24] A smaller market size (and correspondingly higher P) would also increase the low quality firms' information rents. By dividing through by V_H, we measure low quality firms' information rents relative to the high quality firms' surplus, which is independent of P, and the entry fee F, relative to the total entry cost $F + K$ of high quality entrants.

the firm's incentive to invest in high quality, so for a given gap in quasi-rents (and associated entry reputation ϕ^e), the final good price P and output Y must adjust to give firms sufficient incentives to invest in high quality.

At the same time, we see that since $V_H(\phi^e) > V_L(\phi^e)$ for all $\phi^e < 1$, the equilibrium production of the final good Y is increasing in the sales subsidy s. The subsidy can thus be used to offset the impact of entry fees on final goods production Y. Without such a subsidy, since whenever $\alpha_H < 1$, $V_H(\phi^e) - V_L(\phi^e)$ approaches zero as ϕ^e approaches one, a regulator using entry fees alone (s fixed) must eventually face a tradeoff in using those fees between raising the average quality of entrants ϕ^e and reducing the size of market Y.

From these considerations, we obtain two main results showing how the use of regulatory tools F and s complements the role of reputation to improve the average quality of entering firms ϕ^e and/or bring the production of the final consumption good Y closer to its full information socially optimal levels.

IV.A. Regulation Based on Both Entry Fees and Subsidies

First, we show that it is possible to choose policies F and s to implement welfare in steady state arbitrarily close to welfare in the full information first best outcome.

Proposition 5 *Regulation based on fees to entry and subsidies to final good sales.*

There exists a combination of F and s that implements a steady-state allocation with $Y = \bar{Y}$ and $\phi_e = 1 - \epsilon$ for any $\epsilon > 0$, with steady-state welfare approaching the full information first best level as ϵ approaches zero.

The result follows directly from the previous discussion: we can implement any entry reputation that is arbitrarily close to the full information level ($\phi^e = 1 - \epsilon$) by setting F so that $V_L(1-\epsilon)/V_H(1-\epsilon) = F/(F+K)$. Next, we set the corresponding subsidy to implement the first-best output level:

$$V_H(1-\epsilon) - V_L(1-\epsilon) = \frac{K}{(1+s)U'(\bar{Y})}.$$

With these values of F and s, low quality firms disappear at the same rate as the entry reputation converges to 1, and the measure of high quality firms in the steady state, the equilibrium consumption of the numeraire good N, and hence the steady-state welfare level all approach their full information first best values.

This result highlights the forces through which entry regulation improves on the laissez-faire benchmark by restoring incentives for quality investments: the entry fee serves to extract the rents of low quality entrants, shifting the quality mix at entry towards more and more high quality firms, while the subsidy serves to offer high quality entrants a sufficient investment incentive. The combination of an upfront fee with a price subsidy introduces de facto backloading of rewards, which aligns private and social marginal values of investment.

Remark on spot transactions and lack of private commitment: If buyer-seller interactions were repeated and long-term contracts between a buyer and seller could be enforced, then the two parties could design an incentive contract guaranteeing that most sellers entering into the contract are high-quality. Such contracts commit to a deferred compensation scheme with low initial prices, and an extra reward to the firm in the future which is based on the performance of its signals. The lemons problem thus manifests itself in combination with lack of commitment to pay anything other than spot prices, and the fact that spot market prices do not offer firms sufficient incentives to invest in high quality. Regulation is thus a remedy to the lack of private commitment power.

Remark on budget balance: We have assumed that the regulator has access to lump sum taxes and transfer to fund the intervention, and have not imposed budget balance between the entry fee and the price subsidy. Budget balance would impose restrictions on how closely one can approximate the unconstrained first best. The government revenues from entry taxes are Fm^e per period, and in the case of implementing an allocation arbitrarily close to the full information first best (i.e. $\phi^e \to 1$), revenues are automatically bounded, since $F \to V_L(1)U'(\bar{Y})$ by free entry and $m^e \to (1-\bar{\omega})\,\bar{Y}/y(1)$. Expenditures on subsidies on the other hand are $sU'(Y)Y$ per period, and s becomes arbitrarily large as $\phi^e \to 1$. Budget balance thus limits the extent to which tax policies can approximate the first-best allocation.[25]

Remark on non-Markov transfers: Our assumption that subsidies are based on transactions rather than the full history of signals for each firm is restrictive. The standard result that a reputation of $\phi = 1$ is an absorbing state implies that $V_L(1) = V_H(1)$, so it is impossible to completely avoid entry by low quality firms. If instead the regulator was able to make transfers based on a firm's full history of signals about quality, then it would be possible to implement an allocation with $\phi^e = 1$. This is not possible with transfers that are Markov in reputation because buyers ignore further signals of quality once $\phi = 1$. Still, the welfare costs of ruling out complete signal contingent transfers are negligible, as simple transaction subsidies and entry fees as the ones discussed above already get us arbitrarily close to the full information benchmark.

IV.B. Regulation Based Only on Entry Fees

Our second main result shows that entry fees alone can improve on the laissez-faire equilibrium. We first provide a representation of the household's steady-state utility as a strictly increasing function of both Y and ϕ^e, in the limit as $\beta \to 1$. Whenever $V_H(\phi^e) - V_L(\phi^e)$ is increasing in the entry reputation for ϕ^e close to $\underline{\phi}$, small positive entry fees will simultaneously increase aggregate production Y and the entry reputation ϕ^e, i.e. there is no trade-off between quality and aggregate production and small entry fees unambiguously increase welfare.

Proposition 6 *Regulation just based on fees to entry ($s = 0$).*

(i) If β is sufficiently close to 1, then a small positive entry fee is unambiguously welfare-enhancing.

(ii) Moreover, such small positive entry fee unambiguously increases both aggregate output Y and entry reputation ϕ^e.

For (i), we first obtain a representation of welfare in terms of Y and ϕ^e in the limiting case where $\beta \to 1$. In this case, aggregate output is given by

$$Y = m^e \left(\phi^e V_H(\phi^e) + (1 - \phi^e) V_L(\phi^e)\right), \tag{18}$$

[25] In addition, if the price subsidies must be funded through distortionary taxes, then the welfare gains from optimal regulation have to be balanced against the welfare costs of tax distortions.

which using the free entry conditions can be represented as $YU'(Y) = m^e(\phi^e K + F)$. The resources of the numeraire good spent on investing in high quality entrants is then given by

$$m^e \phi^e K = YU'(Y) \frac{\phi^e K}{\phi^e K + F} \tag{19}$$

and steady-state welfare is

$$U(Y) + 1 - YU'(Y) \frac{\phi^e K}{\phi^e K + F}. \tag{20}$$

Therefore, it is immediate that any $F > 0$ that results in an increase in Y unambiguously increases welfare. But the result goes a step further in proving that even with an immediate tradeoff between increasing ϕ^e and reducing Y, the former always dominates the welfare calculus. This follows from the observation that the good firms' free entry condition imposes an upper bound on the potential output loss due to the entry distortion. The welfare gain from a small positive entry fee ($\phi^e > \phi^e_{lf}$) is

$$U(Y) - U(Y_{lf}) - YU'(Y) \frac{\phi^e K}{\phi^e K + F} + Y_{lf} U'(Y_{lf}).$$

From the good firms' entry conditions, we have $U'(Y_{lf}) = K/V_H(\phi^e_{lf}) > K/V_H(\phi^e) = K/(K+F) \cdot U'(Y)$, which bounds the rate at which an entry fee reduces output. Introducing this bound, and combining with the fact that $U(Y) - U(Y_{lf}) > U'(Y)(Y - Y_{lf})$, due to concavity of $U(\cdot)$ we then obtain that the welfare gain from a small positive entry fee is at least

$$U'(Y)\left(Y \frac{F}{\phi^e K + F} - \frac{F}{K+F} Y_{lf}\right).$$

But when F is sufficiently close to 0, and Y sufficiently close to Y_{lf}, this expression must be strictly positive.

For (ii) we simply need to check that $V_H(\phi^e) - V_L(\phi^e)$ is increasing in the entry reputation for ϕ^e close to $\underline{\phi}$. In the Single trial version, this is immediate since

$$V_H(\phi^e) - V_L(\phi^e) = (\alpha_H - \alpha_L)\frac{\beta\bar{\omega}}{1-\beta\bar{\omega}}y(\phi^g(\phi^e)), \tag{21}$$

and $y(\phi^g(\phi^e))$ is increasing in ϕ^e (unless $\alpha_L = 0$, in which case $\phi^g(\phi^e) = 1$ and output does not change with ϕ^e). Because the high quality firm is more likely to receive a good signal and continue with positive profits, it also has to gain more at the margin from a better entry reputation.

The argument is more involved for the Bernoulli trials version. Here, we consider the thought experiment of comparing the laissez-faire benchmark against an intervention that sets the entry reputation $\phi^e = \phi^g(\underline{\phi}) > \underline{\phi}$, equivalent to the occurrence of a single good signal relative to $\underline{\phi}$. From the Bellman equations at $\underline{\phi}$, we have

$$(1-\beta\bar{\omega})\,V_H(\underline{\phi}) = \bar{\omega}\left(y\left(\underline{\phi}\right) + \beta\alpha_H\left(V_H(\phi^g(\underline{\phi})) - V_H(\underline{\phi})\right)\right),$$

i.e. the (positive) flow value of a high type firm at the exit threshold corresponds to the (negative) flow profits plus the expected flow value from obtaining a good signal. Similarly, for a low type firm, we have

$$(1-\beta\bar{\omega})\,V_L(\underline{\phi}) = \bar{\omega}\left(y\left(\underline{\phi}\right) + \beta\alpha_L V_L(\phi^g(\underline{\phi}))\right) = 0.$$

Due to free entry, the current flow profits $y\left(\underline{\phi}\right)$ exactly offset the low type's value of a positive signal. Combining these equations, we obtain an expression for the change in $V_H(\phi^e) - V_L(\phi^e)$ from moving the entry reputation to $\phi^g(\underline{\phi})$:

$$\bar{\omega}\beta\alpha_H\left(V_H(\phi^g(\underline{\phi})) - V_L(\phi^g(\underline{\phi})) - V_H(\underline{\phi})\right) = (1-\beta\bar{\omega})\,V_H(\underline{\phi}) + \bar{\omega}y\left(\underline{\phi}\right)\left(\frac{\alpha_H}{\alpha_L} - 1\right),$$

which is positive whenever

$$(1-\beta\bar{\omega})\,V_H(\underline{\phi}) > -\bar{\omega}y\left(\underline{\phi}\right)\left(\frac{\alpha_H}{\alpha_L}-1\right). \tag{22}$$

Intuitively speaking, the exit condition for low types equates the current flow losses to the low type's expected value of a good signal. But then the high-types flow value is proportional to the difference between the option values of good signals, which maps into the difference in value functions when α_H is close to α_L (e.g. in any continuous time limit, in which a given positive signal carries little information). But since the high-types' flow value must be strictly positive, so must be the difference in the value of a good signal, and hence we have that a small positive intervention raising the entry reputation to $\phi^g(\underline{\phi})$ increases output.

In the symmetric Bernoulli trials case that we consider, $V_H(\phi) - V_L(\phi)$ is increasing in ϕ for all ϕ sufficiently close to $\underline{\phi}$. If ϕ is sufficiently large, however, a tradeoff between quality and production eventually emerges. Thus, the optimal entry fee is positive but remains finite. Such a tradeoff also emerges eventually in the Single Trial version of our model, once ϕ^e gets sufficiently large such that $\phi^b(\phi^e) > \underline{\phi}$. Condition (22) further suggests that this scenario is more broadly representative of our model, e.g. output is at first increasing in ϕ^e, whenever the flow of signals is sufficiently informative relative, but any given positive signal carries little information, and a tradeoff between quality and aggregate production must eventually emerge, whenever $\alpha_H < 1$ and $\lim_{\phi\to 1} V_L(\phi) = \lim_{\phi\to 1} V_H(\phi) = V_H(1)$.

Remark on Bernoulli trials with asymmetric signals: With the exception of Propositions 4 and 6(ii), none of our results or proofs made use of the restriction to symmetric signals that we introduced earlier; they therefore extend directly to arbitrary signal structures with Bernoulli trials. To discuss the role of asymmetries for these two remaining results, we consider two tractable versions of our model with Bernoulli trials with asymmetric signals. First, in the *bad news* case, the probability that the high quality firm receives a bad signal is zero, but such signal is realized with positive probability for low quality firms in each period ($\alpha_H = 1 > \alpha_L > 0$). Second, in the *good news* case, the probability that the low quality firm receives a good signal is zero, but such signal is realized with positive probability for high quality firms ($1 > \alpha_H > \alpha_L = 0$).

In Appendix C, we provide closed form solutions for the value functions of these two cases, and show that proposition 4 also holds, even though the original proof requires some adjustments. Interesting differences emerge with regards to part (ii) of Proposition 6, however. In the bad news case the difference between the value functions $V_H(\phi) - V_L(\phi)$ increases in ϕ for all ϕ greater than the exit threshold and, hence, there is no tradeoff for the regulator between raising the entry reputation ϕ^e and the stationary output of the final consumption good Y when choosing an entry fee. Moreover, as $\lim_{\phi\to 1} V_L(\phi) < \lim_{\phi\to 1} V_H(\phi)$, regulation can implement an entry reputation $\phi^e = 1 - \epsilon$ for arbitrarily small ϵ with a bounded entry fee F.

Condition (22), however, does not help us in the good news case in which $\alpha_L = 0$, i.e. even a single good signal remains highly informative of the firms' quality. In this case, there is an immediate tradeoff emerging between entry reputation and aggregate output. Nevertheless, part (i) of our proposition still applies so optimal entry fees remain positive. The impact of F on Y is potentially ambiguous because F and ϕ^e has two effects on the composition of high and low quality firms that operate in the stationary equilibrium. Holding fixed low quality firms' exit decisions, an increase in the entry reputation increases the ratio of high quality to low quality firms overall, which lowers the investment of the numeraire good required to produce a given level of the final consumption good Y. However, in equilibrium, the continuation decision of low quality firms must adjust to keep the exit reputation at $\underline{\phi}$. That is, an increase in the entry reputation also lowers the rate at which low quality firms exit and therefore increases their life expectancy at entry. This indirect effect of F on average quality through the life expectance of low quality entrants accounts for the difference between the two cases we consider: with bad news, the life expectancy of low quality entrants is independent of ϕ^e, while with good news, the effect of delayed exit by low quality firms becomes very strong.

Remark on the impact of F on numeraire consumption and welfare: The same considerations also complicate accounting for the impact of regulation on the consumption of N and therefore make welfare calculations more subtle. The exception is the limiting case of $\beta \to 1$, where we can make use of the steady-state equivalence between the net present value of new entrants and the cross-sectional distribution of currently active firms to arrive at the simple characterization of numeraire good consumption above.

Remark on the welfare impact of regulation: The magnitude of the impact of regulation on welfare is non-monotonic in the precision of signals. When the precision of signals goes to zero, entry fees do not increase Y much, since the difference between value functions is negligible. However, since the production of the final good is very small, the marginal welfare gain can still be important. At the other extreme, when the precision of signals goes to infinity, there is not much room for improvement on the market outcome to be achieved through regulation, since this outcome is already close to the unconstrained first best. This suggests that regulatory policies are most effective in improving the outcome of a market with spot prices when the precision of signals is intermediate.

Remarks on other regulatory tools: Naturally, a regulator can use other regulatory tools, in addition to entry fees, to increase welfare. For example, if a regulator can observe that a firm remains active, then it can also offer operation subsidies or impose taxes on active firms. Such subsidies or taxes may be helpful in discouraging low-quality firms from continuing operations, or disproportionately rewarding high-quality firms to remain active. Depending on this trade-off, operational taxes or subsidies may complement entry fees in increasing welfare.

Policies that subsidize variables more likely to be experienced by high-quality firms, such as age, or that punish variables more likely to be experienced by low-quality firms, such as exit, can also be exploited. Even though such variables are only imperfect signals of reputations, their incidence may be influenced through the design of the tax policy (e.g. low-quality firms may decide to continue longer if they expect to obtain benefits from age-contingent subsidies), and may be not as easily manipulable, the regulator can design a wide array of policy combinations to impact welfare. Our model offers a simple framework for analyzing the impact of such policies.

V. Conclusions

Most businesses in virtually all countries require licenses to operate, adding substantial costs to the creation of new businesses. The Pigouvian rationale in favor of such regulation is typically based on a concern for the quality of products offered by the entering firms. Yet in practice most entry regulations seem unrelated to explicit concerns about the quality of products offered by these new entrants.

In this paper we have developed a general equilibrium model in which firm entry, growth and exit are all driven by uncertainty and learning about the firms' product quality, and used this model to analyze the impact of entry regulation on product quality, overall production and welfare. Our analysis offers a theoretical foundation for the Pigouvian argument: through a combination of entry fees and price subsidies, a regulator can improve on the laisser-faire outcome with spot market transactions: entry fees discourage firms from entering with low quality products, and through the subsidy a regulator can align the entering high quality firms' incentives with social surplus. Even when the entry fee is the only regulatory tool available, imposing such a fee is strictly welfare improving, and typically does not result in a distortion of market prices and output levels, at least when the fee is small.

In our model, we have assumed that quality is fixed on entry, yet in many respects maintaining quality is an ongoing concern. An important direction for future research is therefore to extend our analysis of regulation to models with ongoing moral hazard. We conjecture that some of our core insights also extend to such a setting: to the extent that the market recognizes and rewards quality imperfectly and with a delay, the laissez-faire incentives for quality investments will be too weak to compensate the seller sufficiently for maintaining a high reputation, so there should again be scope for regulatory interventions that appropriately backload rewards. But the required transfer schemes are likely significantly more complex, and fall outside the scope of what we can solve for at this time.[26]

[26]See, for example, Marvel and McCafferty (1984), Maksimovic and Titman (1991), and, more recently, Board and Meyer-ter-Vehn (2013) for theoretical developments along similar lines.

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A. Proofs of Propositions

We begin with two lemmas that will be used in the subsequent equilibrium characterization and uniqueness results.

Lemma 1: *Suppose that $\phi^c(\cdot)$ is non-decreasing. Then, $V_i(\cdot)$ is non-decreasing in ϕ, and strictly increasing whenever $\omega_i(\phi) > 0$. Moreover, $V_H(\phi) \geq V_L(\phi)$, and $V_H(\phi) > V_L(\phi)$, whenever $\omega_L(\phi) > 0$.*

Proof: Whenever $\phi^c(\cdot)$ is non-decreasing, the Bellman operator $\mathcal{T}_i$:

$$(\mathcal{T}_i V)(\phi) = \max_{\omega \in [0,\overline{\omega}]} \omega \left\{ y(\phi) + \beta \alpha_i V(\phi^g(\phi^c(\phi))) + \beta(1-\alpha_i) V\left(\phi^b(\phi^c(\phi))\right) \right\}$$

preserves continuity and monotonicity, and represents a contraction. By standard arguments, it therefore has a unique fixed point $V_i(\cdot)$ which is continuous and non-decreasing in ϕ. Moreover, whenever $\omega_i(\phi) > 0$, we have, for $\phi' > \phi$ that

$$\begin{aligned} V_i(\phi') - V_i(\phi) \quad &\geq \quad \omega_i(\phi)\{y(\phi') - y(\phi) + \beta\alpha_i[V_i(\phi^g(\phi^c(\phi'))) - V_i(\phi^g(\phi^c(\phi)))] \\ &\quad + \beta(1-\alpha_i)\left[V_i\left(\phi^b(\phi^c(\phi'))\right) - V_i\left(\phi^b(\phi^c(\phi))\right)\right]\right\} \\ &\geq \quad \omega_i(\phi)(y(\phi') - y(\phi)) > 0, \end{aligned}$$

and

$$(\mathcal{T}_H V_L)(\phi) - V_L(\phi) \geq \omega_L(\phi)\beta(\alpha_H - \alpha_L)\left(V_L(\phi^g(\phi^c(\phi))) - V_L\left(\phi^b(\phi^c(\phi))\right)\right)$$

Since $\omega_L(\phi) > 0$, it follows that $V_L(\phi^g(\phi^c(\phi))) > V_L\left(\phi^b(\phi^c(\phi))\right)$, and therefore $V_H(\phi) > (\mathcal{T}_H V_L)(\phi) > V_L(\phi)$.■

Lemma 2: *Consider interim belief functions $\phi^c(\cdot)$, $\hat{\phi}^c(\cdot)$, $\widetilde{\phi}^c(\cdot)$, such that $\widetilde{\phi}^c(\phi) \leq \phi^c(\phi) \leq \hat{\phi}^c(\phi)$ for all ϕ. Suppose further that $\hat{\phi}^c(\cdot)$ and $\widetilde{\phi}^c(\cdot)$ are non-decreasing in ϕ. Then the corresponding value functions $V_i(\cdot)$, $\hat{V}_i(\cdot)$, and $\widetilde{V}_i(\cdot)$ satisfy $\widetilde{V}_i(\phi) \leq V_i(\phi) \leq \hat{V}_i(\phi)$, for all ϕ.*

Proof: Let T_i, $\hat{T}_i$ and $\widetilde{\mathcal{T}}_i$ denote the Bellman operators associated with interim belief functions $\phi^c(\cdot)$, $\hat{\phi}^c(\cdot)$, $\widetilde{\phi}^c(\cdot)$, such that $\widetilde{\phi}^c(\phi) \leq \phi^c(\phi) \leq \hat{\phi}^c(\phi)$ for all ϕ. We wish to show that the fixed point for T_i is bounded by the fixed points for $\hat{T}_i$ and $\widetilde{\mathcal{T}}_i$. For a given monotonic function V, we have

$$\begin{aligned} \left(\hat{\mathcal{T}}_i \hat{V}_i\right)(\phi) &= \max_{\omega \in [0,\overline{\omega}]} \omega \left\{ y(\phi) + \beta\alpha_i \hat{V}_i\left(\phi^g\left(\hat{\phi}^c(\phi)\right)\right) + \beta(1-\alpha_i)\hat{V}_i\left(\phi^b\left(\hat{\phi}^c(\phi)\right)\right)\right\} \\ &\geq \max_{\omega \in [0,\overline{\omega}]} \omega \left\{ y(\phi) + \beta\alpha_i \hat{V}_i\left(\phi^g\left(\phi^c(\phi)\right)\right) + \beta(1-\alpha_i)\hat{V}_i\left(\phi^b\left(\phi^c(\phi)\right)\right)\right\} \\ &= \left(\mathcal{T}_i \hat{V}_i\right)(\phi), \end{aligned}$$

and therefore $\hat{V}_i \geq \left(\mathcal{T}_i \hat{V}_i\right)(\phi)$, and since T_i is monotonic, $\left\{\mathcal{T}_i^{(k)} \hat{V}_i(\cdot)\right\}$ is a decreasing sequence of functions, converging to $V_i(\cdot)$. Therefore $V_i(\phi) \leq \hat{V}_i(\phi)$, for all ϕ. The argument to show that $\widetilde{V}_i(\phi) \leq V_i(\phi)$ for all ϕ is analogous.■

It follows from this lemma that we can bound equilibrium beliefs and value functions by focusing on the highest and lowest interim belief functions.

Proof of Proposition 1: Follows directly from the construction of value functions and interim beliefs provided in the text.■

Proof of Proposition 2: Using lemma 2, we can bound value functions at any equilibrium by using the "extreme" interim beliefs. Any interim beliefs for which $\phi^c(\phi) \geq \phi$ for all $\phi \in [0,1]$ result in a firm value function $V_i(\cdot) \geq V_i^*(\cdot)$, where $V_i^*(\cdot)$ is the value function characterized by the Bellman equation with $\phi^c(\phi) = \phi$. Since for all $\phi \geq \underline{\phi}$, $V_H(\phi) \geq V_L(\phi) \geq V_L^*(\phi) > 0$, it follows that at any equilibrium, low- (and high-) quality firm with $\phi > \underline{\phi}$ strictly prefer to continue, implying that $\omega_H(\phi) = \omega_L(\phi) = \bar{\omega}$ and $\phi^c(\phi) = \phi$.

Next, we show that at any equilibrium, $\phi^c(\phi) \leq \underline{\phi}$ for any $\phi \leq \underline{\phi}$: if to the contrary, $\phi^c(\phi) > \underline{\phi} > \phi$ then the value of low quality firms would be strictly positive (since the value is bounded below by $V_L(\phi^c(\phi)) \geq V_L^*(\phi^c(\phi)) > 0$). But then, $\omega_L(\phi) = \bar{\omega}$ from which it follows that necessarily $\phi^c(\phi) \leq \phi$ - a contradiction. Therefore, using lemma 2, we bound value functions at any equilibrium by using the "extreme" interim beliefs $\phi^c(\phi) = \phi$ and $\phi^c(\phi) = \max\left\{\underline{\phi}, \phi\right\}$. Since they both sustain the same value $V_L^*(\cdot)$ for low quality firms, it follows that the low-type value function is uniquely determined in any equilibrium.

Moreover, given the value function for low types, it follows that whenever $\phi^c(\phi) < \underline{\phi}$, $y(\phi^c(\phi)) + \beta\alpha_L V_L^*(\phi^g(\phi^c(\phi))) < y(\underline{\phi}) + \beta\alpha_L V_L^*(\phi^g(\underline{\phi})) = 0$ so low quality firms have a strict incentive to exit. For each ϕ, we are thus left with two possibilities: either the low quality firms mix over exit in such a way that $\phi^c(\phi) = \underline{\phi}$, or the high-quality firms and low quality firms both exit with probability 1, so that continuation is an off-equilibrium event.

Now consider the high quality firm's incentives. In any equilibrium, let $\hat{\phi} \leq \underline{\phi}$ denote the lowest reputation level such that any firm with reputation $\phi > \hat{\phi}$ continue with positive probability, and $\phi^c(\phi) = \min\left\{\underline{\phi}, \phi\right\}$ and $V_i(\phi) = V_i(\underline{\phi})$. For a high quality firm with reputation $\phi \leq \hat{\phi}$, s.t. $\phi^g(\phi) > \hat{\phi}$, we have that

$$\begin{aligned} y(\phi^c(\phi)) + \beta\alpha_H V_H(\phi^g(\phi^c(\phi))) &\geq y(\phi) + \beta\alpha_H V_H(\phi^g(\phi)) \\ &\geq y(\phi) + \beta\alpha_H V_H(\underline{\phi}) \geq y(\phi) + \beta\alpha_H V_H^*(\underline{\phi}), \end{aligned}$$

where the first inequality follows from "continuation signals quality", the second from the fact that $\phi^g(\phi) > \hat{\phi}$ and therefore $\phi^c(\phi^g(\phi)) = \underline{\phi}$, and the third uses the bound on $V_H(\cdot)$ implied by lemma 2. If $y(0) + \beta\alpha_H V_H^*(\underline{\phi}) \geq 0$, this last expression is strictly positive, implying that any firm slightly below $\hat{\phi}$ must strictly prefer to continue - contradicting the definition $\hat{\phi}$, if $\hat{\phi} > 0$. Therefore, it must be the case that $\hat{\phi} = 0$, so that interim beliefs are uniquely determined for all strictly positive reputation levels.■

Proof of Proposition 3: Follows immediately from $V_H(\phi) < V_H(1) = \frac{\bar{\omega}}{1-\beta\bar{\omega}} y(1)$.■

Proof of Proposition 4: We prove this result for the single signal version of the model, and defer the proof for the Bernoulli trials version with symmetric signals to appendix B and with two cases of asymmetric signals (good news and bad news cases) to appendix C.

Write $\phi^g(\phi) \equiv \phi^g(\phi; \frac{\alpha_H}{\alpha_L})$ to make explicit the dependence of the posterior belief on the informativeness of a good signal. From Bayes Rule we have that $\phi^g(\phi; \frac{\alpha_H}{\alpha_L})$ is strictly increasing in ϕ and in $\frac{\alpha_H}{\alpha_L}$ for all $0 < \phi < 1$. Let $\phi = F(\phi^g; \frac{\alpha_H}{\alpha_L})$ denote the inverse of $\phi^g(\phi; \frac{\alpha_H}{\alpha_L})$. Notice that F is strictly increasing in $0 < \phi^g < 1$ and strictly decreasing in $\frac{\alpha_H}{\alpha_L}$. Recall in addition that in the single signal version, $y(\phi^g(\phi^e_{lf}; \frac{\alpha_H}{\alpha_L})) > 0 > y(\phi^e_{lf})$. From the free entry conditions, we have that ϕ^e_{lf} and Y_{lf} satisfy

$$\begin{aligned}(\alpha_H - \alpha_L)\frac{\beta\overline{\omega}}{1-\beta\overline{\omega}}y(\phi^g(\phi^e_{lf}; \frac{\alpha_H}{\alpha_L})) &= \frac{K}{U'(Y_{lf})} \\ \alpha_L\frac{\beta\overline{\omega}}{1-\beta\overline{\omega}}y(\phi^g(\phi^e_{lf}; \frac{\alpha_H}{\alpha_L})) &= -y(\phi^e_{lf})\end{aligned}$$

It is useful to rewrite these conditions in terms of ϕ^g as

$$(\alpha_H - \alpha_L)\frac{\beta\overline{\omega}}{1-\beta\overline{\omega}}y(\phi^g) = \frac{K}{U'(Y_{lf})} \tag{23}$$

$$\alpha_L\frac{\beta\overline{\omega}}{1-\beta\overline{\omega}}y(\phi^g) = -y(F(\phi^g; \frac{\alpha_H}{\alpha_L})) \tag{24}$$

In equation (24), the LHS is increasing in α_L and in ϕ^g, while the RHS is decreasing in ϕ^g and α_L, and increasing in α_H. Therefore, the solution $\phi^g\left(\alpha_H, \alpha_L\right)$ to (24) is strictly increasing in α_H and strictly decreasing in α_L. But then an increase in α_H or a decrease in α_L both increase the LHS of equation (23), which implies that Y_{lf} must also increase.■

Proof of Proposition 5: Follows directly from the arguments in the main text.■

Proof of Proposition 6: Part (i) We prove the result by showing that the result holds in the limit as $\beta \to 1$, for ϕ^e sufficiently close to $\underline{\phi}$. The result then applies for $\beta \to 1$ sufficiently small because of continuity. Steady-state welfare can be written as $U\left(Y\right) + 1 - m^e\phi^e K$. In the limit as $\beta \to 1$, we have that $Y = m^e\left(\phi^e V_H\left(\phi^e\right) + \left(1-\phi^e\right)V_L\left(\phi^e\right)\right)$, due to the equivalence (in the absence of time discounting) between an entering firms' expected discounted future profits over time, and the cross-section of firms' current profits at any given period. But then, steady-state welfare takes the form

$$\begin{aligned} W(Y,\phi^e) &= U(Y) + 1 - \frac{\phi^e K}{\phi^e V_H(\phi^e) + (1-\phi^e) V_L(\phi^e)} \cdot Y \\ &= U(Y) + 1 - \frac{\phi^e K}{\phi^e K + F} \cdot U'(Y) Y. \end{aligned}$$

Now, for a small entry fee F, we have

$$W(Y,\phi^e) - W\left(Y_{lf}, \phi^e_{lf}\right) = U(Y) - \frac{\phi^e K}{\phi^e K + F} \cdot U'(Y) Y - U(Y_{lf}) + U'(Y_{lf}) Y_{lf}$$

Clearly if $Y \geq Y_{lf}$, $W(Y,\phi^e) > W\left(Y_{lf}, \phi^e_{lf}\right)$, and welfare is strictly higher with a small entry fee. Suppose therefore that $Y < Y_{lf}$. Now, notice that for $\phi^e > \phi^e_{lf}$, we have

$$K + F = U'(Y) V_H(\phi^e) \geq U'(Y) V_H\left(\phi^e_{lf}\right) = \frac{U'(Y)}{U'(Y_{lf})} K,$$

and therefore $U'(Y_{lf}) \geq K/(F+K) \cdot U'(Y)$, and

$$\begin{aligned} W(Y,\phi^e) - W\left(Y_{lf}, \phi^e_{lf}\right) &\geq U(Y) - U(Y_{lf}) - U'(Y)\left(\frac{\phi^e K}{\phi^e K + F} Y - \frac{K}{K+F} Y_{lf}\right) \\ &\geq U'(Y)(Y - Y_{lf}) - U'(Y)\left(\frac{\phi^e K}{\phi^e K + F} Y - \frac{K}{K+F} Y_{lf}\right) \\ &= U'(Y)\left(\frac{F}{\phi^e K + F} Y - \frac{F}{K+F} Y_{lf}\right) \end{aligned}$$

but since $\phi^e K + F < K + F$ (strictly also when $F = 0$), it follows that it is always possible to find a $Y_{lf} - Y > 0$ sufficiently close to 0 that is implemented with a F sufficiently close to 0 (by continuity) such that $Y/(\phi^e K + F) > Y_{lf}/(K+F)$, and therefore $W(Y,\phi^e) > W\left(Y_{lf}, \phi^e_{lf}\right)$.

Part (ii): This is immediate for the Single signal case, given equation (21). We show in Appendix B, for Bernoulli trails with symmetric signals, that equation (22) holds.■

B. Bernoulli Trials with Symmetric Signals

In this section we provide analytical solutions to the symmetric signals cases of the Bernoulli trials version of our model. For tractability, we express value functions in terms of a transformed variable

$$l(\phi) = \frac{1-\phi}{\phi},$$

i.e. the odds ratio that a firm is of low type (from now on "low type odds ratio"). The firm's reputation is $\phi(l) = 1/(1+l)$. We denote the value functions V_i, per period output y and continuation strategies ω_i in terms of this low type odds ratio, l. The updating from continuation, good and bad signals respectively takes the form:

$$l^c(l) = l \cdot \omega_L(l)/\omega_H(l), \quad l^g(l) = l \cdot \alpha_L/\alpha_H \quad \text{and} \quad l^b(l) = l \cdot (1-\alpha_L)/(1-\alpha_H).$$

We let l^e and $\underline{l}$ denote the low type odds ratio at entry and at the exit point for low type firms. With the transformation, $0 < l^e \leq \underline{l}$.

With symmetric signals, $\alpha_H = 1 - \alpha_L$. Let $\eta = \alpha_L/\alpha_H \in (0,1)$, then $\alpha_H = 1/(1+\eta)$ and $\alpha_L = \eta/(1+\eta)$. Because of symmetry good and bad signals exactly off-set each other in the firm's reputation: $l^g\left(l^b(l)\right) = l$. Starting from an entry reputation of $\underline{l}$, the firm's reputation therefore increases or decreases along a discrete ladder: $l \in \{l_0, l_1, ...\}$, where $l_k = \underline{l} \cdot \eta^k$. The first time the firm accumulates strictly more bad signals than good signals, a low quality firm exits with positive probability to compensate for the corresponding loss in reputation.

Equilibrium value functions: The value function of a firm of type $i \in \{H, L\}$ solves the Bellman equation

$$V_i(l_k) = \bar{\omega}\left(y(l_k) + \beta\alpha_i V_i(l_{k+1}) + \beta(1-\alpha_i)V_i(l_{k-1})\right)$$

together with $V_L(l_{-1}) = V_L(l_0) = \bar{\omega}(y(l_0) + \beta\alpha_L V_L(l_1)) = 0$ for low-quality firms and $V_H(l_{-1}) = V_H(l_0) > 0$ for high-quality firms. Now, define $\Omega_i(k) \equiv \bar{\omega}\beta\alpha_i V_i(l_k) - \chi V_i(l_{k-1})$, where χ is chosen so that $\bar{\omega}\beta\alpha_i/\chi = (1-\chi)/(\bar{\omega}\beta(1-\alpha_i))$, or

$$\chi = \frac{1}{2} - \sqrt{\frac{1}{4} - (\bar{\omega}\beta)^2 \frac{\eta}{(1+\eta)^2}}.$$

The Bellman equation can then be rewritten as

$$\begin{aligned} \bar{\omega}\beta\alpha_i V_i(l_{k+1}) - \chi V_i(l_k) + \bar{\omega} y(l_k) &= (1-\chi) V_i(l_k) - \bar{\omega}\beta(1-\alpha_i) V_i(l_{k-1}), \\ \text{or } \Omega_i(k) &= (\bar{\omega}\beta\alpha_i/(1-\chi))(\Omega_i(k+1) + \bar{\omega} y(l_k)). \end{aligned}$$

Next, define $b(\eta) \equiv \bar{\omega}\beta\alpha_H/(1-\chi)$, and notice that

$$b(\eta) \equiv \frac{\bar{\omega}\beta\alpha_H}{1-\chi} = \frac{2\bar{\omega}\beta}{1+\eta+\sqrt{(1+\eta)^2 - 4(\bar{\omega}\beta)^2\eta}} < \bar{\omega}\beta,$$

so that $\bar{\omega}\beta\alpha_L/(1-\chi) = \eta b(\eta) < b(\eta) < 1$. It follows that

$$\Omega_i(k) = \bar{\omega}\sum_{t=0}^{\infty}\left(\frac{\bar{\omega}\beta\alpha_i}{1-\chi}\right)^{t+1} y\left(\underline{l}\eta^{k+t}\right).$$

Since $\Omega_L(0) = (\bar{\omega}\beta\alpha_L - \chi) V_L(\underline{l}) = 0$, the exit point $\underline{l}$ solves $0 = \bar{\omega}\sum_{t=0}^{\infty}(\eta b(\eta))^{t+1} y(\underline{l}\eta^t)$, or equivalently

$$\frac{-y(0)}{y(1)-y(0)} = (1-\eta b(\eta))\sum_{t=0}^{\infty}(\eta b(\eta))^t \frac{1}{1+\underline{l}\cdot\eta^t}.$$

as $y(l\eta^t) = y(1)\frac{1}{1+l\cdot\eta^t} - y(0)\left(1-\frac{1}{1+l\cdot\eta^t}\right) = (y(1)-y(0))\frac{1}{1+l\cdot\eta^t} - (-y(0))$.

In the same vein, the value of high types at the exit point $\underline{l}$ is given by

$$\begin{aligned} V_H(\underline{l}) &= \frac{\Omega_H(0)}{\bar{\omega}\beta\alpha_H - \chi} = \frac{1}{\bar{\omega}\beta\alpha_H - \chi}\bar{\omega}\sum_{t=0}^{\infty}(b(\eta))^{t+1} y\left(\underline{l}\eta^t\right) \\ &= \frac{\bar{\omega}}{1-\bar{\omega}\beta}\left(\frac{1-\bar{\omega}\beta}{\bar{\omega}\beta\alpha_H - \chi} b(\eta)\right)\sum_{t=0}^{\infty}(b(\eta))^t y\left(\underline{l}\eta^t\right) \\ &= \frac{\bar{\omega}(y(1)-y(0))}{1-\bar{\omega}\beta}(1-b(\eta))\sum_{t=0}^{\infty}(b(\eta))^t\left(\frac{1}{1+\underline{l}\eta^t} - \frac{-y(0)}{y(1)-y(0)}\right) \end{aligned}$$

where the last step follows from adding and subtracting $1 - b(\eta)$:

$$\begin{aligned}\frac{1-\bar{\omega}\beta}{\bar{\omega}\beta\alpha_H-\chi}b(\eta) &= 1-b(\eta)+\frac{(1-\bar{\omega}\beta)\bar{\omega}\beta\alpha_H-(\bar{\omega}\beta\alpha_H-\chi)(1-\chi)+(\bar{\omega}\beta\alpha_H-\chi)\bar{\omega}\beta\alpha_H}{(\bar{\omega}\beta\alpha_H-\chi)(1-\chi)} \\ &= 1-b(\eta)+\frac{\chi(1-\chi)-(\bar{\omega}\beta)^2\alpha_H(1-\alpha_H)}{(\bar{\omega}\beta\alpha_H-\chi)(1-\chi)}=1-b(\eta).\end{aligned}$$

Continuity and Differentiability: Finally, consider $l \notin \{l_0, l_1, ...\}$. Starting from any $l \in [l_0, l_1]$, a firm's reputation evolves on a discrete grid $\{l, l\eta, l\eta^2, ...\}$ until the first instant the number of bad signals exceeds the number of good signals. At this point, a low-type firm exits with positive probability, and the continuing firms' reputation shifts to $\underline{l}$. Since it is always optimal for low type firms to exit the first time the firm experiences more bad than good signals (and for high types never to exit), we can use this strategy to define the firm's expected payoffs $V_L(\cdot)$ and $V_H(\cdot)$, and since per period payoffs are decreasing, continuous and differentiable in l, it follows that these value functions are decreasing, continuous and differentiable over $l \in [l_0, l_1]$ (with left- and right-continuous differentiability at the boundaries).

The same argument extends to any interval $l \in [l_{k-1}, l_k]$ with exit by low types the first time that the number of bad signals exceeds the number of good signals by k. We therefore have that the value functions are decreasing, continuous and piece-wise differentiable, with kinks possible along the grid $l \in \{l_0, l_1, ...\}$.

Monotonicity of Laissez-Faire outcome (Proposition 4): Write $V_L(l;\eta)$, $V_H(l;\eta)$ and $\underline{l}(\eta)$ to indicate the dependence of value functions and the exit threshold w.r.t. the informational parameter η. The high quality firms' free entry condition is $K/U'(Y_{lf}) = V_H(\underline{l}(\eta);\eta)$. Since $K/U'(Y_{lf})$ is increasing in Y_{lf}, we therefore wish to show that $V_H(\underline{l}(\eta);\eta)$ is decreasing in η. The condition defining the exit threshold $\underline{l}(\eta)$ can be written as $F(\eta, \underline{l}) = 0$, where

$$\begin{aligned} F(\eta, \underline{l}) &= \underline{l}(1-\eta b(\eta)) \sum_{t=0}^{\infty} (\eta b(\eta))^t \left(\frac{1}{1+\underline{l}\cdot\eta^t} - \frac{-y(0)}{y(1)-y(0)} \right) \\ &= (1-\eta b(\eta)) \sum_{t=0}^{\infty} (b(\eta))^t \left(1 - \frac{1}{1+\underline{l}\cdot\eta^t} \right) - \frac{-y(0)}{y(1)-y(0)} \underline{l} \\ &= \Psi(\eta) \left(1 - (1-b(\eta)) \sum_{t=0}^{\infty} (b(\eta))^t \frac{1}{1+\underline{l}\cdot\eta^t} \right) - \frac{-y(0)}{y(1)-y(0)} \underline{l}, \end{aligned}$$

where

$$\Psi(\eta) = \frac{1-\eta b(\eta)}{1-b(\eta)}.$$

The high type's value is

$$\begin{aligned} V_H(\underline{l};\eta) &= \frac{\bar{\omega}(y(1)-y(0))}{1-\bar{\omega}\beta} \left\{ (1-b(\eta)) \sum_{t=0}^{\infty} (b(\eta))^t \frac{1}{1+\underline{l}\eta^t} - \frac{-y(0)}{y(1)-y(0)} \right\} \\ &= \frac{\bar{\omega}(y(1)-y(0))}{1-\bar{\omega}\beta} \left\{ \frac{y(1)}{y(1)-y(0)} - \frac{-y(0)}{y(1)-y(0)} \frac{\underline{l}}{\Psi(\eta)} \right\}, \end{aligned}$$

from the equation that determines $\underline{l}(\eta)$. Then, $V_H(\underline{l}(\eta);\eta)$ is decreasing in η if and only if $\underline{l}(\eta)/\Psi(\eta)$ is increasing in η, or equivalently $\underline{l}'(\eta)/\underline{l}(\eta) > \Psi'(\eta)/\Psi(\eta)$. By the Implicit function Theorem, $\underline{l}'(\eta) = -F_\eta(\eta,\underline{l})/F_{\underline{l}}(\eta,\underline{l})$, so that this condition can be re-written as

$$F_\eta(\eta, \underline{l}(\eta)) + \frac{\Psi'(\eta)}{\Psi(\eta)} \underline{l}(\eta) F_{\underline{l}}(\eta, \underline{l}(\eta)) > 0,$$

where

$$F_{\underline{l}}(\eta, \underline{l}(\eta)) = -\frac{-y(0)}{y(1)-y(0)} + (1-\eta b(\eta)) \sum_{t=0}^{\infty} (\eta b(\eta))^t \frac{1}{(1+\underline{l}(\eta)\eta^t)^2} < 0, \text{ and}$$

$$
\begin{aligned}
F_\eta\left(\eta,\underline{l}\left(\eta\right)\right) &= \Psi'\left(\eta\right)\left(1-\left(1-b\left(\eta\right)\right)\sum_{t=0}^{\infty}\left(b\left(\eta\right)\right)^t\frac{1}{1+\underline{l}\left(\eta\right)\eta^t}\right)\\
&\quad+\left(1-\eta b\left(\eta\right)\right)\sum_{t=0}^{\infty}\left(b\left(\eta\right)\right)^t\frac{\underline{l}\left(\eta\right)\cdot t\cdot\eta^{t-1}}{\left(1+\underline{l}\left(\eta\right)\eta^t\right)^2}\\
&\quad-\Psi\left(\eta\right)b'\left(\eta\right)\sum_{t=0}^{\infty}\left(t\left(b\left(\eta\right)\right)^{t-1}-\left(t+1\right)\left(b\left(\eta\right)\right)^t\right)\frac{1}{1+\underline{l}\left(\eta\right)\eta^t}\\
&= \frac{\Psi'\left(\eta\right)}{\Psi\left(\eta\right)}\frac{-y(0)}{y(1)-y(0)}\underline{l}\left(\eta\right)+\frac{1}{\eta}\left(1-\eta b\left(\eta\right)\right)\sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{t\underline{l}\left(\eta\right)}{\left(1+\underline{l}\left(\eta\right)\eta^t\right)^2}\\
&\quad-\Psi\left(\eta\right)b'\left(\eta\right)\sum_{t=0}^{\infty}\left(t+1\right)\left(b\left(\eta\right)\right)^t\left(\frac{1}{1+\underline{l}\left(\eta\right)\eta^{t+1}}-\frac{1}{1+\underline{l}\left(\eta\right)\eta^t}\right)\\
&= \frac{\Psi'\left(\eta\right)}{\Psi\left(\eta\right)}\frac{-y(0)}{y(1)-y(0)}\underline{l}\left(\eta\right)+b\left(\eta\right)\left(1-\eta b\left(\eta\right)\right)\sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{\left(t+1\right)\underline{l}\left(\eta\right)}{\left(1+\underline{l}\left(\eta\right)\eta^{t+1}\right)^2}\\
&\quad-\Psi\left(\eta\right)b'\left(\eta\right)\left(1-\eta\right)\sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{\left(t+1\right)\underline{l}\left(\eta\right)}{\left(1+\underline{l}\left(\eta\right)\eta^{t+1}\right)\left(1+\underline{l}\left(\eta\right)\eta^t\right)}
\end{aligned}
$$

From these derivatives, we obtain

$$
\begin{aligned}
&F_\eta\left(\eta,\underline{l}\left(\eta\right)\right)+\frac{\Psi'\left(\eta\right)}{\Psi\left(\eta\right)}\underline{l}\left(\eta\right)F_{\underline{l}}\left(\eta,\underline{l}\left(\eta\right)\right)\\
=\;&\frac{\Psi'\left(\eta\right)}{\Psi\left(\eta\right)}\left(1-\eta b\left(\eta\right)\right)\sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{\underline{l}\left(\eta\right)}{\left(1+\underline{l}\left(\eta\right)\eta^t\right)^2}+b\left(\eta\right)\left(1-\eta b\left(\eta\right)\right)\sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{\left(t+1\right)\underline{l}\left(\eta\right)}{\left(1+\underline{l}\cdot\eta^{t+1}\right)^2}\\
&-\Psi\left(\eta\right)b'\left(\eta\right)\left(1-\eta\right)\sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{\left(t+1\right)\underline{l}\left(\eta\right)}{\left(1+\underline{l}\cdot\eta^{t+1}\right)\left(1+\underline{l}\cdot\eta^t\right)}.
\end{aligned}
$$

Now, notice that $b'\left(\eta\right)\leq 0$, and

$$
\begin{aligned}
\sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{t+1}{\left(1+\underline{l}\cdot\eta^{t+1}\right)^2} &> \sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{t+1}{\left(1+\underline{l}\cdot\eta^{t+1}\right)\left(1+\underline{l}\cdot\eta^t\right)}>\sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{t+1}{\left(1+\underline{l}\cdot\eta^t\right)^2}\\
&= \sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\sum_{s=0}^{\infty}\left(\eta b\left(\eta\right)\right)^s\frac{1}{\left(1+\underline{l}\cdot\eta^{t+s}\right)^2}\\
&> \sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{1}{\left(1+\underline{l}\cdot\eta^t\right)^2}\sum_{s=0}^{\infty}\left(\eta b\left(\eta\right)\right)^s\\
&= \frac{1}{1-\eta b\left(\eta\right)}\sum_{t=0}^{\infty}\left(\eta b\left(\eta\right)\right)^t\frac{1}{\left(1+\underline{l}\cdot\eta^t\right)^2}.
\end{aligned}
$$

In addition,

$$\begin{aligned}\Psi'(\eta) &= \frac{-b(\eta)(1-b(\eta))+b'(\eta)(1-\eta)}{(1-b(\eta))^2}, \text{ or} \\ \frac{\Psi'(\eta)}{\Psi(\eta)}(1-\eta b(\eta)) &= \frac{b'(\eta)(1-\eta)}{1-b(\eta)} - b(\eta).\end{aligned}$$

But then it follows immediately that

$$\begin{aligned}& F_\eta(\eta, \underline{l}(\eta)) + \frac{\Psi'(\eta)}{\Psi(\eta)}\underline{l}(\eta) F_{\underline{l}}(\eta, \underline{l}(\eta)) \\ > & \left\{\frac{\Psi'(\eta)}{\Psi(\eta)}(1-\eta b(\eta)) + b(\eta) - \Psi(\eta)\frac{1-\eta}{1-\eta b(\eta)} b'(\eta)\right\} \sum_{t=0}^{\infty} (\eta b(\eta))^t \frac{\underline{l}(\eta)}{(1+\underline{l}(\eta)\eta^t)^2} = 0.\end{aligned}$$

Desirability of Positive Entry Fees (Proposition 6): From equation (22), it suffices to check that

$$V_H(\underline{l}) \geq -\frac{\bar{\omega}}{1-\bar{\omega}\beta} y(\underline{l})\left(\frac{1}{\eta}-1\right).$$

Using the high type's value function, we rewrite this condition as

$$(1-b(\eta))\sum_{t=0}^{\infty}(b(\eta))^t y(\underline{l}\eta^t) \geq -y(\underline{l})\frac{1-\eta}{\eta}$$

which we rearrange as

$$\begin{aligned}(1-b(\eta))\sum_{t=1}^{\infty}(b(\eta))^t y(\underline{l}\eta^t) &\geq -y(\underline{l})\left((1-b(\eta)) + \frac{1}{\eta} - 1\right), \text{ or} \\ b(\eta)(1-b(\eta))\sum_{t=0}^{\infty}(b(\eta))^t y(\underline{l}\eta^{t+1}) &\geq -y(\underline{l})\frac{1}{\eta}(1-\eta b(\eta)).\end{aligned}$$

From the low type's free entry condition, we have

$$-y(\underline{l}) = \sum_{t=1}^{\infty}(\eta b(\eta))^t y(\underline{l}\eta^t) = \eta b(\eta)\sum_{t=0}^{\infty}(\eta b(\eta))^t y(\underline{l}\eta^{t+1})$$

and therefore the condition becomes

$$(1-b(\eta))\sum_{t=0}^{\infty}(b(\eta))^{t}\,y\left(\underline{l}\eta^{t+1}\right)\geq(1-\eta b(\eta))\sum_{t=0}^{\infty}(\eta b(\eta))^{t}\,y\left(\underline{l}\eta^{t+1}\right),$$

which holds for all $\eta<1$.

C. Bernoulli Trials with Asymmetric Signals

In this section we provide analytical solutions to the two extreme cases that we consider of the Bernoulli trials version of our model with asymmetric signals.

III.A. Bad News Signals

In the bad news case, $1=\alpha_H>\alpha_L$, i.e. a low quality firm is revealed with positive probability to be of low quality. A firms' reputation monotonically increases with time after entry, until the arrival of a bad signal results in a discrete drop of the firm's reputation to 0. Therefore, there is no exit by low quality firms until a bad signal realizes. The low type odds ratio of a firm that entered t periods ago is $l^e\cdot\alpha_L^t$. Let $v_i(l)$ denote the value of a type i firm that stays in the market until a low signal realizes. A low type firm has a survival probability of $\bar{\omega}\alpha_L$ per period, so

$$\begin{aligned}
v_L(l) &= \bar{\omega}\left(y(l)+\beta\bar{\omega}\alpha_L v_L(l\alpha_L)\right)=\bar{\omega}\sum_{t=0}^{\infty}(\beta\bar{\omega}\alpha_L)^t\,y\left(l\cdot\alpha_L^t\right)\\
&= \bar{\omega}\left(y(1)-y(0)\right)\sum_{t=0}^{\infty}(\beta\bar{\omega}\alpha_L)^t\left(\frac{1}{1+l\cdot\alpha_L^t}-\frac{-y(0)}{(y(1)-y(0))}\right)
\end{aligned}$$

since $y(\phi)=\phi y(1)+(1-\phi)y(0)$ and $\phi=1/(1+l)$. A high type firm instead has a survival probability of $\bar{\omega}$:

$$\begin{aligned}
v_H(l) &= \bar{\omega}\left(y(l)+\beta\bar{\omega}v_H(l\alpha_L)\right)=\bar{\omega}\sum_{t=0}^{\infty}(\beta\bar{\omega})^t\,y\left(l\cdot\alpha_L^t\right)\\
&= \bar{\omega}\left(y(1)-y(0)\right)\sum_{t=0}^{\infty}(\beta\bar{\omega})^t\left(\frac{1}{1+l\cdot\alpha_L^t}-\frac{-y(0)}{(y(1)-y(0))}\right)
\end{aligned}$$

Both of these value functions are strictly decreasing, continuous and differentiable in l. Moreover $\lim_{l\to\infty} v_L(l) = \bar{\omega}y(0)/(1-\beta\bar{\omega}\alpha_L) < 0 < \lim_{l\to 0} v_L(l) = \bar{\omega}y(1)/(1-\beta\bar{\omega}\alpha_L)$, so there exists a unique value $\underline{l}$ such that $v_L(\underline{l}) = 0$. Moreover, from the application of lemma 1, it follows that $v_H(l) > v_L(l)$ for all $l \leq \underline{l}$. We have thus established that the value functions $V_L(l) = \max\{0, v_L(l)\}$ and $V_H(l) = v_H(\max\{l, \underline{l}\})$, along with exit threshold $\underline{l}$, sustain the equilibrium that is characterized in propostion 1 and shown to be unique when "Continuation implies quality" under proposition 2.

Moreover, notice that the two value functions evaluated at $\underline{l}$, $v_H(\underline{l})$ and $v_L(\underline{l})$, take exactly the same form as in the symmetric signals case, with $b(\eta)$ set equal to $\beta\bar{\omega}$ and η set equal to α_L. Therefore, it is straight-forward to check that the proofs for propositions 4 and 6 extend directly to the bad news scenario. Therefore, $v_H(\underline{l}(\alpha_L);\alpha_L)$ and Y_{lf} are decreasing in α_L (proposition 4), and targeting the entry reputation to $l\alpha_L$ strictly increases output and welfare (proposition 6).

In fact, proposition 6 can even be strengthened by noting that

$$\frac{v_H(l)-v_L(l)}{\bar{\omega}(y(1)-y(0))} = \sum_{t=0}^{\infty}(\beta\bar{\omega})^t\left(1-\alpha_L^t\right)\left(\frac{1}{1+l\cdot\alpha_L^t} - \frac{-y(0)}{(y(1)-y(0))}\right)$$

and

$$\frac{v_H'(l)-v_L'(l)}{\bar{\omega}(y(1)-y(0))} = -\sum_{t=0}^{\infty}(\beta\bar{\omega})^t\left(1-\alpha_L^t\right)\frac{\alpha_L^t}{\left(1+l\cdot\alpha_L^t\right)^2} < 0,$$

i.e. any decrease in l (increase in entry reputation) unambiguously raises aggregate output - i.e. in the bad news model, the tradeoff between quality and market size never materializes.

III.B. Good News Signals

In the good news case, $\alpha_H > \alpha_L = 0$, i.e. a high quality firm is revealed with positive probability to be of high quality. A firms' reputation monotonically decreases with time after entry, until either a good signal results in a discrete jump of the firms' reputation to 1, or low quality firms start to exit with a positive rate once their value reaches zero. In this case, by Bayes'rule a firm with reputation l that doesn't experience a good signal has its reputation updated to $l/(1-\alpha_H) > l$, unless $l/(1-\alpha_H) > \underline{l}$, in which case its reputation remains at $\underline{l}$ (and low-quality firms exit with positive probability).

Equilibrium value functions: Since the value of low-quality firms always remains at 0 at the laissez-faire benchmark, we have that $y(\underline{l}) = 0$, or $\frac{1}{1+\underline{l}} = -y(0)/(y(1)-y(0))$, i.e. static flow profits of a newly entering firm must be zero. For $l < \underline{l}$, a low-quality firm's value function is

$$\begin{aligned} V_L(l) &= \bar{\omega}(y(1)-y(0)) \sum_{t=0}^{\infty} (\beta\bar{\omega})^t \max\left\{0, \frac{1}{1+l\cdot(1-\alpha_H)^{-t}} - \frac{-y(0)}{y(1)-y(0)}\right\} \\ &= \bar{\omega}(y(1)-y(0)) \sum_{t=0}^{\infty} (\beta\bar{\omega})^t \max\left\{0, \frac{1}{1+l\cdot(1-\alpha_H)^{-t}} - \frac{1}{1+\underline{l}}\right\}. \end{aligned}$$

At the laissez-faire benchmark, the high quality firms then earn zero profits until a good signal about their quality results in a jump of their reputation to 1. Hence the high quality firms' value function at the laissez-faire benchmark solves

$$V_H(\underline{l}) = \bar{\omega}\beta(\alpha_H V_H(1) + (1-\alpha_H) V_H(\underline{l})) = \frac{\bar{\omega}\beta\alpha_H}{1-\bar{\omega}\beta(1-\alpha_H)} V_H(1),$$

where $V_H(1) = \frac{\bar{\omega}}{1-\beta\bar{\omega}} y(1)$. For $l < \underline{l}$, the high quality firm earns positive flow profits until its reputation reaches $\underline{l}$ in the absence of a good signal. Hence for $l < \underline{l}$, its value function is

$$\begin{aligned} V_H(l) &= \bar{\omega}(y(l) + \beta\alpha_H V_H(1) + \beta(1-\alpha_H) V_H(l/(1-\alpha_H))) \\ &= \bar{\omega}(y(1)-y(0)) \sum_{t=0}^{\infty} (\beta\bar{\omega}(1-\alpha_H))^t \max\left\{0, \frac{1}{1+l\cdot(1-\alpha_H)^{-t}} - \frac{1}{1+\underline{l}}\right\} \\ &\quad + \frac{\bar{\omega}\beta\alpha_H}{1-\bar{\omega}\beta(1-\alpha_H)} V_H(1). \end{aligned}$$

Clearly these two value functions are continuous in l, strictly decreasing, and piece-wise differentiable, with kinks possible only if $l \in \left\{\underline{l}, \underline{l}(1-\alpha_H), \underline{l}(1-\alpha_H)^2, ...\right\}$.

Monotonicity of Y_{lf} (Proposition 4): We need to show that $V_H(\underline{l}(\alpha_H), \alpha_H)$ is increasing in the arrival probability of the good signal α_H. But this result follows immediately from the fact that $\bar{\omega}\beta\alpha_H/(1-\bar{\omega}\beta(1-\alpha_H))$ is strictly increasing in α_H.

Desirability of positive entry fees (Proposition 6): Notice that

$$\begin{aligned} V_H(l) - V_L(l) &= \frac{\bar{\omega}\beta\alpha_H}{1-\bar{\omega}\beta(1-\alpha_H)} V_H(1) \\ &\quad -\bar{\omega}(y(1)-y(0)) \sum_{t=0}^{\infty} (\beta\bar{\omega})^t \left(1-(1-\alpha_H)^t\right) \max\left\{0, \frac{1}{1+l\cdot(1-\alpha_H)^{-t}} - \frac{1}{1+\underline{l}}\right\}, \end{aligned}$$

i.e. the gap $V_H(l^e) - V_L(l^e)$, and hence the equilibrium market size, is strictly increasing in l, or decreasing in entry reputation ϕ^e. On the other hand, still applies so for sufficiently small entry fees, the positive impact on entrant quality more than offsets the negative impact of smaller equilibrium market size.